Insights into the Theory of Relativity. Part III. Dynamic Relativist. 1.-Matter and Energy Equivalence.
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Abstract

The equivalence between matter and energy, and the variation of the classic mass with velocity, are developed. We highlight that the classical prove of the cited equivalence, arise from the fact of the existence of energy, force, and acceleration into the context of an inertial referential frame. It shows up a contradiction with the concept of inertial referential frame at rest.

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1 Introduction

The relativist dynamic, is a part extremely innovative of the physics.

Here, basically, we will deal with matters about equivalence between matter and energy and the variation of the mass with velocity.

We show the prove of the equivalence between mass and energy in the classic theory [1]. We point out that acceleration, forces, etc... show up in inertial reference frames at rest, (in the theory of relativity in the deduction of the equivalence between matter and energy) .

Therefore, it is shown up a contradiction with the context of inertial referential frame at rest.

Thereby it is necessary to consider that, in the especial theory of relativity, we are dealing with an initial equilibrium rest state and a final equilibrium rest state, in the analysis of the dynamic relativist (where come up forces and accelerations, between the initial and final state).

The purpose of this article is to develop the theme of the transformation between mass and energy as an introduction for afterward analysis.

[1]This prove appear in many books and articles about the theory of relativity, without thinking into the original articles of Einstein.

2 Magnitudes generalized in the minkowskian spaces (tertravectors)

In the special theory of relativity, the physical magnitudes are to be tensorial magnitudes.

Usually these magnitudes are scalars, vectors, or tensors in a 4 dimensions minkowskian space. Vectors in a minkowskian space are called tetravectors.

These tensorial magnitudes have a physical reality into the theory of relativity, into the context of minkowskian space L_4.

Among the most important magnitudes defined in this way,we have:

scalars: *interval of universe* ds; *mass at rest* m_0; *proper time* τ

Vectorial (tetravectors): *vector position* $\overrightarrow{dx}; \overrightarrow{dx} \in L_4$; *velocity generalized* $\overrightarrow{U}$; *generalized linear momentum* $\overrightarrow{\Pi}$; *generalized force* $\overrightarrow{\Phi}$

Some magnitudes into the context of minkowskian space L_4, , are composed of other, for example $\overrightarrow{U} = \frac{\overrightarrow{dx}}{d\tau}$.

By the way, all the magnitudes defined into L_4, are called *generalized magnitudes.*

In L_4, only *generalized magnitudes* have physical sense because of they have to be defined in the space-time, (or minkowskian space L_4).

3 Magnitudes defined in classical galilean spaces.

Generalized magnitudes are in connection with *non relativistic magnitudes* of classical physics.

generalized magnitudes act in the space-time. The *non relativistic magnitudes* act into the galilean context.

The *non relativistic magnitudes* are fitted to the spatial or temporal components of the *generalized magnitudes.*

4 Matter enegy equivalence in the Special Theory of Relativity.

In the special theory of relativity the *interval of universe* ds is:

$$ds^2 = -c^2 dt^2 + \sum_{i=1}^{3} (dx^i)^2$$

$$ds^2 = -c^2dt^2 + \sum_{i=1}^{3}(dx^i)^2 = -c^2d\tau^2$$

where τ the *proper time*. Therefore

$$d\tau = dt\sqrt{1 - \frac{v^2}{c^2}}$$

$$\boxed{\gamma = \sqrt{1 - \frac{v^2}{c^2}}}$$

where

$$\overrightarrow{v} = (\tfrac{dx^1}{dt}, \tfrac{dx^2}{dt}, \tfrac{dx^3}{dt}) \quad ; \quad v = \sqrt{\overrightarrow{v}^2}$$

$$d\tau = \gamma dt$$

We have

$$\overrightarrow{U} = \tfrac{d\overrightarrow{x}}{d\tau} = (\tfrac{dx^0}{d\tau}, \tfrac{dx^1}{d\tau}, \tfrac{dx^2}{d\tau}, \tfrac{dx^3}{d\tau})$$

where $\overrightarrow{U}$ is the the *generalized velocity*
Then

$$\overrightarrow{U} = \tfrac{d\overrightarrow{x}}{d\tau} = (\tfrac{c}{\gamma}, \tfrac{dx^1}{\gamma dt}, \tfrac{dx^2}{\gamma dt}, \tfrac{dx^3}{\gamma dt})$$

namely

$$\overrightarrow{U} = (\tfrac{c}{\gamma}, \tfrac{\overrightarrow{v}}{\gamma})$$

Let be now $\overrightarrow{\Pi}$, the *generalized linear momentum* and m_0 a scalar called *mass at rest* [2].

$$\overrightarrow{\Pi} = m_0\overrightarrow{U} = (\tfrac{m_0}{\gamma}c, \tfrac{m_0}{\gamma}\overrightarrow{v}) = (mc, m\overrightarrow{v})$$

where m has to be the mass of the matter in the classic theory.

$$\boxed{m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

[2] We consider it as a relativist generalization of the mass. Actually it arises from the fact that m_0 is the mass when $\overrightarrow{v} = \overrightarrow{0}$

$\overrightarrow{\Phi}$ is the *force generalized.*

$$\overrightarrow{\Phi} = \frac{d\overrightarrow{\Pi}}{d\tau} = \frac{1}{\gamma}\left(\frac{d(mc)}{dt}, \frac{d(m\overrightarrow{v})}{dt}\right)$$

Taking into account that $\overrightarrow{U}^2 = -c^2$ it is verified

$$\overrightarrow{\Pi}^2 = m_0^2.\overrightarrow{U}^2 = -c^2 m_0^2 \tag{1}$$

Then

$$\overrightarrow{\Pi}.\frac{d\overrightarrow{\Pi}}{d\tau} = 0 \quad \text{therefore} \quad \overrightarrow{\Pi}.\overrightarrow{\Phi} = 0$$

$$\frac{1}{\gamma}\left(\frac{d(mc)}{dt}, \frac{d(m\overrightarrow{v})}{dt}\right).(mc, m\overrightarrow{v}) = 0$$

It is clear that

$$m\overrightarrow{v}\frac{d(m\overrightarrow{v})}{dt} - mc\frac{d(mc)}{dt} = 0$$

$$\overrightarrow{v}.\overrightarrow{f} = \frac{dE}{dt} = \frac{d(mc^2)}{dt}$$

$$\boxed{\delta E = \delta(mc^2)}$$

$\overrightarrow{E}$, $\overrightarrow{f}$ and $\overrightarrow{p}$ are the *energy*, *force* and *linear momentum* in the classical basics theory.

The formula of equivalence between mass and energy is:

$$\boxed{\delta E = \delta(mc^2)}$$

This is the famous formula of Einstein for inertial reference frames in the special relativity theory.

5 Linear momentum and energy.

With that been said, $\delta E = \delta(mc^2)$ or rather $E = mc^2$, it is clear that

$$\overrightarrow{\Pi} = (\tfrac{E}{c}, \overrightarrow{p})$$

Then:

$$\overrightarrow{\Pi}^2 = -\tfrac{E^2}{c^2} + \overrightarrow{p}^2$$

Keeping into memory 1

$$\overrightarrow{\Pi}^2 = -c^2 m_0^2$$

Therefore

$$\tfrac{E^2}{c^2} - \overrightarrow{p}^2 = c^2 m_0^2$$

It is worthwhile to highlight that the energy of a particle with *linear momentum* $\overrightarrow{p}$, and rest mass m_0 has to be:

$$E = c\sqrt{\overrightarrow{p}^2 + c^2 m_0^2}$$

In this way it is possible to define the energy of the $foton$ (rest mass $m_0 = 0$):

$$E = c\overrightarrow{p}$$

ANNEXES

A Notations, symbols and terminology

Vectors are symbolized with over right arrow.

Tensors and endomorphisms stand for bold or normal uppercase letters.

The matrix of components of an endomorphism, tensor, etc.. is shown closing inside parenthesis the symbol of this endomorphism, tensor, etc.. . For example $(\mathbf{T})$ stands for the matrix of components of $\mathbf{T}$. $(\mathbf{g}_{\alpha\beta})$ is a matrix which elements are $\mathbf{g}_{\alpha\beta}$.

However for convenience we omit parenthesis when specified in order for using symbols more easily.

The two vectors scalar product $\overrightarrow{\mathbf{x}}$ and $\overrightarrow{\mathbf{y}}$ is symbolized by $\mathbf{G}(\overrightarrow{\mathbf{x}}, \overrightarrow{\mathbf{y}})$ where $\mathbf{G}$ is the metric tensor. Also is symbolized by $\overrightarrow{\mathbf{x}}.\overrightarrow{\mathbf{y}}$.

Subscripts are symbolized by lower case greek or latin letters, saving λ and μ that are used to denote invariants.

Usually E_2 symbolize a 2-dim euclidean space, L_2 a 2-dim vectorial lorentzian space and L_n a n-dim vectorial lorentzian space. Meanwhile we do not know if the space is lorentzian L_n or euclidean E_n, we symbolize these spaces with symbol $\mathbb{L}_n$ In general if it is not established if the space is lorentzian or eucledian we will use the blackboard bold letter [3] to represent the space.

$T^\sharp$ is de G-adjoint endomorphism of T. T^t is the transposed endomorphism of T. In regard to the called *endomorphism associated to a tensor* it is necessary to make clear that the components of the mentioned endomorphism are those of the mixed components of the tensor.

[3] For example L in blackboard bold letter is $\mathbb{L}$

References

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[2] F.Sánchez; Insights into theory of relativity. Part II. Loretz Transformation (2018). relativityworkshop.com.

[3] R. Penrose ; The road to reality (Jonathan Cape; London 2004); Chapters 14 and 17. It is about some concepts on the affine space.

[4] A. A. Logunov ; Lectures on Relativity Theory and Gravitation. URSS Publishers 1998; spanish edition). Lectures 5 and 6. Highlight the relevance that Poincaré found out in the development of special relativity. It is really important to single out that the basics of classic Theory of Relativity are in the transformations that leave invariant the Maxwell equations.

[5] F.Sánchez; Structures of the Skew-adjoint Endomorphisms and Some Peculiarities of Electromagnetic Field.(2014). Relativityworkshop.com.

[6] Gregory L. Naber; The geometry of Minkowski spacetime ;(Springer Verlag 1982). It is a full analysis of the skew-adjoint nature of electromagnetic field

[7] Roger A. Horn; Matrix Analysis;(Cambridge University Press 1985). It is an appealing text about matrix analysis really useful as theorical background here.

[8] Marsall C. Pease III; Methods of Matrix Algebra; Academic Press 1965; pag 223 and next; Adjoint operator . This book works on covariant coordenates. Here we work on contravariant coordinates. Therefore here $F_i^\sharp = GF_i^t G^{-1}$ $i = 1, 2$. ; In general we work on contravariant coordenates unless otherwise specified. It is a theorical background essential.