About the space-time generated by electromagnetic field.

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Abstract

We throw a new light on the basic principles of the Theory of Relativity. The structure of the vectorial lorentzian space-time, that is, its signature (-1,1,1,1), and the $c\ factor$ (velocity of light) are deduced from the skew-symmetric features of electromagnetic tensorial fields. In general, the space is endowed of a metric, but without thinking of the signature of this metric.

In short, we attain the next outcomes:

The electro-magnetic field generates his own minkowskian space-time namely the Lorentz 4 dimensions vectorial space.

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Contents

1 Introduction

In this article I intend to prove the next:

The vectorial lorentzian space-time that supports the electromagnetic field is inferred from the electromagnetic field skew-adjoint features and from the 4-dimensions [1] of space-time endowed of a metric (regarding of its signature). [2]

We prove this on the basis of the skew-adjoint characteristics of electromagnetic tensor, and on the 4-dimensions of the space. In the beginning, we only know the metric [3], but is not specified how is his signature. We rule out other models of space-time that arise into our relativistic scheme and are inferred in this paper, but do not fit with the actual physical reality (see Annex B2).

That said, we will construct the 4 dimensional vectorial Lorentz space-time. [4]

We present this paper describing firstly how is generated the space-time on the basis of features of the electromagnetic field, without going into details. We will explain the details in the annexes **A**, **B**, **C** and **D**.

The electromagnetic field (together with gravitational field) is the field predominant in the macrocosm. Therefore the space-time associated to this field, is also predominant in the universe known

[1] Into the macrocosm, in the context of measurements gotten with three measurement rods and a clock, the electromagnetic tensor field has to be formulated by means of algebraic equations, into a 4 dimensions space. As we will see later on, annihilating polynomial (of the endomorphism associated to the tensor of electromagnetic field) has to be of grade 4 maximum.

[2] From now on, we will use the expression $space - time$. The *time* in the expression $space - time$, has to come up into a term of the metric. This term has to involve the time. For the moment it has nothing to do with the signature.

[3] The specific metric will arise from the fact that the endomorphism associated to the tensorial field is skew-adjoint.

[4] It is essential to know the article "Structures of the Skew-adjoint Endomorphisms and Some Peculiarities of Electromagnetic Field" (on-line article in the web site www.relativityworkshop.com); or [3] ; see scheme into Annexe **A**. It is necessary this theoretical background to understand this paper. We shall refer to the cited article some times. See also [4].

2 Deduction of the vectorial lorentzian features of space-time

Any endomorphism associated with a tensor, has his characteristic polynomial. This polynomial can or cannot be a minimal polynomial. Actually it is an annihilating polynomial into a vectorial (lorentzian or not lorentzian) space.

The endomorphism F associated to the tensor of electromagnetic field, acts in a 4 dimensions vectorial space. Then his annihilating polynomial is:

$$P_4(F) \equiv (F^4 + a_3 F^3 + a_2 F^2 + a_1 F + a_0 I) = 0 \tag{1}$$

That is $P_4(F)$ is an annihilating polynomial of F.

In Annex **B** it is proved that the polynomial 1 is transformed into:

$$P_4(F) \equiv (F^2 + \epsilon\lambda^2 I)(F^2 + \eta\mu^2 I) \tag{2}$$

$\epsilon = \pm 1$; $\eta = \pm 1$ [5]

In Annex **B2** are analyzed feasible options, on the basis of the ϵ and η values, and on the basis of a **regular endomorphisms** (see Annex **B1**).

Basically we have three options:

a) $\epsilon = -1$; $\eta = +1$
b) $\epsilon = -1$; $\eta = -1$
c) $\epsilon = +1$; $\eta = +1$

Option a) is the option we select because of it allows us to set up the special theory of relativity that we know.

Option b) is of interest to develop it apart. It shows some interesting outcomes that can add something new to the basic theory, but is beyond the scope of our purpose here. It is better to fit it to a bitemporal model.

[5] The prove can be seen also in Section 3.2 of "Structures of the Skew-adjoint Endomorphisms and Some Peculiarities of Electromagnetic Field" article on-line in the web site www.relativityworkshop.com or see [3])

Option c) is ruled out as it is not compatible with the known special relativity.

Options a), b) and c), are analyzed in Annexe **B2**.

In these options we only abide by **regular endomorphisms**.

3 Derivation of the endomorphism associated to the electromagnetic tensor field .

Henceforth we abide by **option a)**, since it is the unique option that fits to the physical reality of the special theory of relativity and it is also the unique option that fits to our relativistic scheme. This is the unique option that we do not rule out.

In Annex **B2** we have proved that the annihilating polynomial 2 is turned into 3

$$P_4(F) \equiv (F - \lambda I)(F + \lambda I)(F^2 + \mu^2 I) \tag{3}$$

It is relevant to highlight that in annexe **B** (and up to now) we generate a space-time regardless of the characteristics of a vectorial lorentzian space.

The next equations system fulfil 3:

$$\begin{aligned} F(\vec{p}) &= \lambda \vec{p} \\ F(\vec{q}) &= -\lambda \vec{q} \\ F(\vec{Y}) &= \mu \vec{Z} \\ F(\vec{Z}) &= -\mu \vec{Y} \end{aligned} \tag{4}$$

Keeping in memory F is skew-adjoint, vectors base $\overrightarrow{\mathbf{p}}, \overrightarrow{\mathbf{q}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}}$ verify (see prove in Annexe **B3**):

$$\overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{q}} = w; \quad \overrightarrow{\mathbf{Y}}^2 = \overrightarrow{\mathbf{Z}}^2 = 1; \quad \overrightarrow{\mathbf{p}}^2 = \overrightarrow{\mathbf{q}}^2 = 0$$

$$\mathbf{p}, \mathbf{q} \perp \mathbf{Y}, \mathbf{Z}; \quad \mathbf{Y} \perp \mathbf{Z}$$

.

Taking as reference $\overrightarrow{\mathbf{p}}, \overrightarrow{\mathbf{q}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}}$, then we have:

$$(\mathbf{F}_\alpha{}^\beta) = \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & +\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix} ; \alpha, \beta = 0, 3 \qquad (5)$$

4 Lorentzian vectorial space-time metric

Here we find out the metric involved by the electromagnetic field F, because we have gotten to know the scalar product of vectors base $\overrightarrow{\mathbf{p}}, \overrightarrow{\mathbf{q}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}}$ (see Annex **B3**):

$$(G_{\alpha\beta}) = \begin{pmatrix} 0 & w & 0 & 0 \\ w & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} ; \alpha, \beta = 0, 3$$

This is an F generated metric, that is, the metric associated to electromagnetic field. Its grammian is $-w^2 < 0$.

As one can easily verify (in accordance with the Sylvester law of inertia), there is a reference system in which the signature is (-1,1,1,1) or (1,-1,-1,-1) , because of the grammian is < 0. Therefore it is the metric of a vectorial Lorentz space-time.

Actually, **it is *a metric* generated by endomorphism F associated to the electromagnetic field tensor in our context of regular endomorphism.**

These outcomes allow us to infer the vectorial lorentzian space-time metric on the basis of skew-adjoint electromagnetic tensor features and of the 4-dimensions nature of space (without thinking into the signature of this space).

Or rather, to infer it, we only have started from these afore mentioned features of electromagnetic field, and of the 4-dimensions nature of space (without thinking into his signature).

In short, we have a 4 dimensional space-time generated by the electromagnetic field F.

It is necessary to single out that the electromagnetic field only acts into its cited generated space-time.

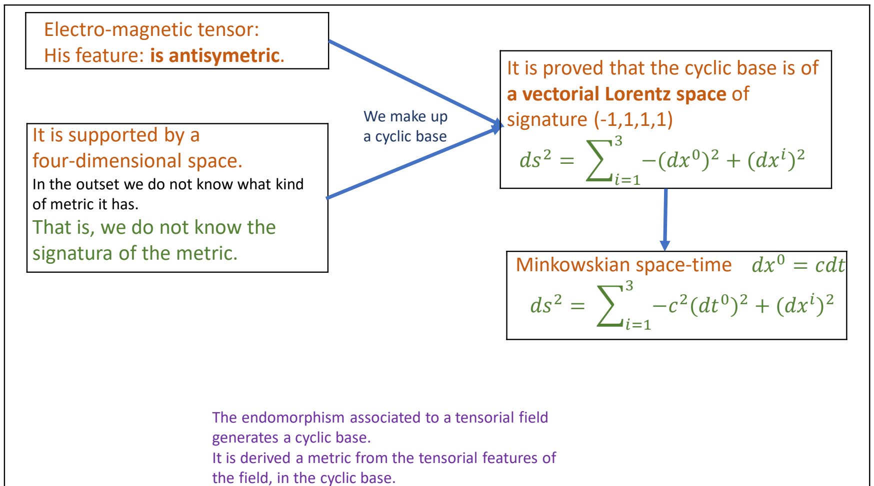

Fig.2

It is necessary to establish that the term $-dx^{0^2}$ of the metric, that is, the term associated to the negative term of the metric, is time dependent. It has to be $-c^2t^2$. [6] Here we abide by that c is a dimensional coupling constant with the dimension of speed. In fact, as dx^i and $ds^2 = \sum dx^{i^2}$; $i = 0, 3$ are longitude measures, dx^0 is also a longitude measure. It has only to do with time. Then c is to be a dimensional constant with the dimension of the velocity.

It is worthwhile to prove that in the case we are dealing here, the time is only involved in the term $-dx^{0^2}$ of the metric, and can not be involved with other positive term as $+dx^{i^2}$. See proof in Annex **D**.

In this way we do not need to postulate a defined metric for space-time, neither the constancy of the light speed, nor a signature for the metric. All of them are derived from the previously cited skew-adjoint electromagnetic tensor features.

[6] In the minskowskian space-time (unlike the lorentzian space-time), it shows up the term *time* and the c factor, that is nothing but the velocity of the light).

5 Principle of coupling of observers.

Actually it is not essential that **the principle of coupling of observers** appears as a principle.

It is based in the fact of two observers that observe the same physical phenomenon. Each observer gets measures of electromagnetic field. These measures are written down as components of a tensor. These components make up a matrix for each observer. For the purpose of both observer can see the same physical phenomenon, they have to reduce their matrices to the same *box of Frobenius.* That is, both matrices are similar, (likewise in the **Principle of coupling of observers**).

6 Conclusions.

¿Are all the fields tensorial fields? It is questionable.

Any way, fields existing in the beginning of the universe could have other features (for example not skew adjoint, but other characteristics). The space-time nature could not be the same.

A Features of electromagnetic field

In Fig.1 we show some outcomes of the above mentioned article [3]. The lorentzian space L_4 is the direct sum of a lorentzian subspace L_2 and an euclidean subspace E_2. We call to the space-time L_4, the **regular Lorentz vectorial space.** We differ this outcome of other kinds of lorentzian spaces or subspaces, called singular. They are shown up in our relativistic scheme.

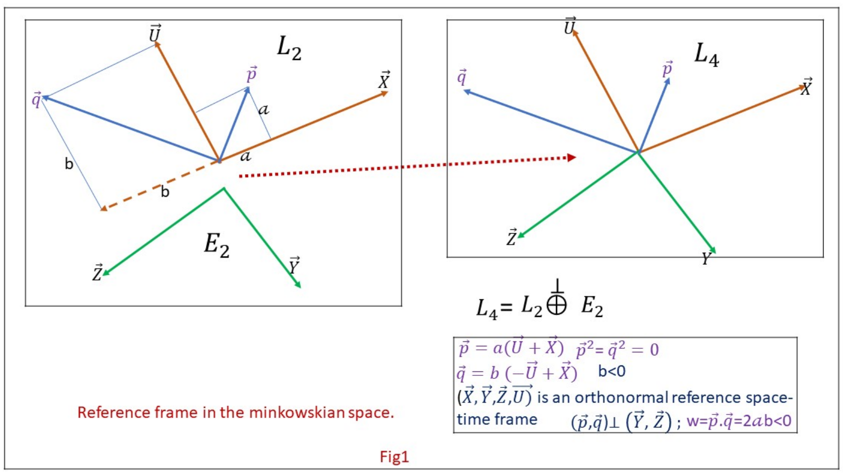

Fig1

This figure is a scheme of the cited article [3].

B Feasible models of space-time in our relativistic scheme.

The most general case of annihilating polynomial in our context of lorentzian space L_4 is:

$$P(F) = F^4 + a_3F^3 + a_2F^2 + a_1F + a_0I \tag{6}$$

We have

$$\forall\overrightarrow{\mathbf{X}}\ ;\ \overrightarrow{\mathbf{X}} \in L_4\ ;\ P(F)\overrightarrow{\mathbf{X}} = 0$$

Then we have.

$$\forall\overrightarrow{\mathbf{X}} \in L_4 \quad ; \quad g(\overrightarrow{\mathbf{X}}, P(F)\overrightarrow{\mathbf{X}}) = 0$$

From previous equations , and taking into account that F^2 and F^4 are self-adjoint endomorphisms, and that F and F^3 are skew-adjoint endomorphisms, we have

$$\forall\overrightarrow{\mathbf{X}} \in L_4 \quad ; a_3g(\overrightarrow{\mathbf{X}}, F^3\overrightarrow{\mathbf{X}}) = 0 \text{ and } a_1g(\overrightarrow{\mathbf{X}}, F\overrightarrow{\mathbf{X}}) = 0$$

6 is reduced to

$$\boxed{P_4(F) \equiv F^4 + a_2F^2 + a_0I = (0)}$$

Summing up, equation $P_4(F) \equiv F^4 + a_2F^2 + a_0I = (0)$ depicts annihilating polynomial on L_4, that concern us.

Here, in the real field, we are only concerned with the factorized polynomial:

$$\boxed{P_4(F) \equiv (F^2 + \epsilon\lambda^2 I)(F^2 + \eta\mu^2 I); \epsilon = \pm 1; \eta \pm 1}$$

As it is easily checked $a_2 = \epsilon\lambda^2 + \eta\mu^2$; $a_0 = \epsilon\eta\lambda^2\mu^2$

B.1 Orthogonality relations into L_4

.

We intend to prove that if

$$\boxed{\lambda \neq 0 \quad \text{or} \quad \mu \neq 0 \quad \text{and} \quad \epsilon\lambda^2 \neq \eta\mu^2}$$

the subspaces $ker(F^2 + \epsilon\lambda^2 I)$ **and** $ker(F^2 + \eta\mu^2 I)$ **are orthogonal.**

$$\forall \overrightarrow{\mathbf{X}} \in ker(F^2 + \epsilon\lambda^2 I), \forall \overrightarrow{\mathbf{Y}} \in ker(F^2 + \eta\mu^2 I)$$

we have

$$\overrightarrow{\mathbf{Y}} F^2 \overrightarrow{\mathbf{X}} = -\epsilon\lambda^2 \overrightarrow{\mathbf{Y}}\overrightarrow{\mathbf{X}}$$
$$\overrightarrow{\mathbf{X}} F^2 \overrightarrow{\mathbf{Y}} = -\eta\mu^2 \overrightarrow{\mathbf{X}}\overrightarrow{\mathbf{Y}}$$

We infer

$$\overrightarrow{\mathbf{X}}.\overrightarrow{\mathbf{Y}}.(\epsilon\lambda^2 - \eta\mu^2) = 0$$

Taking into account the foregoing assumptions we have:

$$\forall \overrightarrow{\mathbf{X}} \in ker(F^2 + \epsilon\lambda^2 I), \forall \overrightarrow{\mathbf{Y}} \in ker(F^2 + \eta\mu^2 I) \qquad \overrightarrow{\mathbf{X}}.\overrightarrow{\mathbf{Y}} = 0$$

and therefore

$$\boxed{ker(F^2 + \epsilon\lambda^2 I) \perp ker(F^2 + \eta\mu^2 I)}$$

Now we prove the intersection $ker(F^2 + \epsilon\lambda^2 I)$ and $ker(F^2 + \eta\mu^2 I)$ is not the null vector $\mathbf{l}$.

We know that $\mathbf{l}^2 = 0$.

According with spectral theorem (see for example [8]), in the intersection $\overrightarrow{\mathbf{l}} \in ker(F^2 + \epsilon\lambda^2 I) \cap \ker(F^2 + \mu^2 I)$, solely can be $\overrightarrow{\mathbf{l}} = \overrightarrow{\mathbf{0}}$.

Any way we prove this intersection is empty, that is $\overrightarrow{\mathbf{l}} = \overrightarrow{\emptyset}$.

If intersection is not empty, then there is a null vector $\overrightarrow{\mathbf{l}}$ that verifies

$$(F^2 + \epsilon\lambda^2 I)\overrightarrow{\mathbf{l}} = 0$$
$$(F^2 + \eta\mu^2 I)\overrightarrow{\mathbf{l}} = 0$$

Subtracting previous equations we have

$$(\epsilon\lambda^2 - \eta\mu^2)\overrightarrow{\mathbf{l}} = \overrightarrow{\mathbf{0}} \tag{7}$$

Taking into account our previous assumptions $\lambda \neq 0$ or $\mu \neq 0$, and $\epsilon\lambda^2 \neq \eta\mu^2$, the foregoing equation 7 involves $\overrightarrow{\mathbf{l}} = \overrightarrow{\mathbf{0}}$ [7]

Summing up

$$\lambda \neq 0 \quad \text{or} \quad \mu \neq 0 \quad \text{and} \quad \epsilon\lambda^2 \neq \eta\mu^2 \tag{8}$$

involves orthogonality between $\ker(A^2 + \epsilon\lambda^2 I)$ and $ker(A^2 + \eta\mu^2 I)$, with zero intersection, that is,

$$\boxed{L_4 \equiv ker(F^2 + \epsilon\lambda^2 I) \overset{\perp}{\oplus} ker(F^2 + \mu^2 I)}$$

We call regular cases or regular endomorphism to the endomorphisms that verify 8 agree with **Annex B**.

B.2 Possible space-time schemes.

On the basis of the mentioned equation (Annex B):

$$P_4(F) \equiv (F^2 + \epsilon\lambda^2 I)(F^2 + \eta\mu^2 I)$$
$$\epsilon = \pm 1 \; ; \; \eta \pm 1$$

we can classify the models of space-time in connection with the ϵ and η values

B.2.1 $\epsilon \neq \eta$

Then, it is equivalent to $\epsilon = -1; \eta = +1$

$$\boxed{P_4(F) \equiv (F^2 - \lambda^2 I)(F^2 + \mu^2 I)}$$

[7]In the case $\epsilon \neq \eta$ is enough $\lambda \neq 0$ or $\mu \neq 0$ to fulfil $L_4 \equiv ker(F^2 + \epsilon\lambda^2 I) \overset{\perp}{\oplus} ker(F^2 + \mu^2 I)$

$$\boxed{P_4(F) \equiv (F-\lambda I)(F+\lambda I)(F^2+\mu^2 I);}$$

This is the known scheme of the special relativity; see [4] and [5].

This scheme is the most suitable because of from him the classical theory of special relativity is developed and it is also the unique option fitted to our relativistic scheme (see [3], [4] and [5]). As we will see in the next, the other schemes are not compatibles.

B.2.2 $\epsilon = +1; \eta = +1$

Equation $P_4(F) \equiv (F^2+\epsilon\lambda^2 I)(F^2+\eta\mu^2 I)$ becomes:

$$\boxed{P_4(F) \equiv (F^2+\lambda^2 I)(F^2+\mu^2 I)}$$

$ker(F^2+\lambda^2 I)$ and $(F^2+\mu^2 I)$ are euclidean.

The demostration can be seen in [3], in *subsection 3.4 Annihilating polynomials* $A^2-\lambda^2 I$ *;* $A^2+\mu^2 I$ *and their invariant subspaces structure ; b).*

Because of the euclidean structure of these subspaces, there are not null vectors and therefore the speed of the light is neither an invariant nor a constant, nor a limit.

For this reason we rule out this scheme.

B.2.3 $\epsilon = -1; \eta = -1$

For in that case, $P_4(F) \equiv (F^2-\epsilon\lambda^2 I)(F^2-\eta\mu^2 I)$ becomes :

$$\boxed{P_4(F) \equiv (F-\lambda I)(F+\lambda I)(F-\mu I)(F+\mu I);}$$

The subspaces invariants are 1 DIM, and are generated by vectors : $\overrightarrow{p}$, $\overrightarrow{q}$, $\overrightarrow{l}$, and $\overrightarrow{m}$, that verifies:

$$\begin{array}{c} F(\overrightarrow{p}) = \lambda\overrightarrow{p} \\ F(\overrightarrow{q}) = -\lambda\overrightarrow{q} \\ F(\overrightarrow{l}) = \mu\overrightarrow{l} \\ F(\overrightarrow{m}) = -\mu\overrightarrow{m} \end{array}$$

For reason of F is a skew-adjoint endomorphism, we have:

$$\overrightarrow{p}F(\overrightarrow{p}) = 0 = \lambda\overrightarrow{p}^2$$
$$\overrightarrow{q}F(\overrightarrow{q}) = 0 = \lambda\overrightarrow{q}^2$$
$$\overrightarrow{l}F(\overrightarrow{l}) = 0 = \mu\overrightarrow{l}^2$$
$$\overrightarrow{m}F(\overrightarrow{m}) = 0 = \mu\overrightarrow{m}^2$$

It is clear that vectors $\overrightarrow{p}$, $\overrightarrow{q}$, $\overrightarrow{l}$ and $\overrightarrow{m}$, are null vectors

Further to that, agree with *appendix B.1*, "orthogonality relations into L_4", we have $\overrightarrow{p}, \overrightarrow{q} \perp \overrightarrow{l}, \overrightarrow{m}$.

On the basis of the fact that two orthogonal null vectors are proportional, it is not hard to see:

$$\overrightarrow{p} = a\overrightarrow{l} = b\overrightarrow{m} \qquad\qquad \overrightarrow{q} = \acute{a}\overrightarrow{l} = \acute{b}\overrightarrow{m}$$

Therefore vectors $\overrightarrow{p}$, $\overrightarrow{q}$, $\overrightarrow{l}$, and $\overrightarrow{m}$ are they all proportional among them. Thereby we have that L_4 **is reduced to a 1-DIM null space**.

Into this context, we also can consider the case of two isotropic cones. This case of two isotropic cones involves a bitemporal model. Likely needs a 5 dimensions space-time. It is beyond of our purpose here. We rule it in this paper.

B.2.4 In short:

The only case that for the moment has physical reality, is the depicted in **B.2.1**.

It is interesting the case **B.2.3**, case of space time reduced to a space of dimension 1, lightlike, that would be studied aside in other paper.

C Einstenian case (real case)

Here, we are working on the base of regular case, that is, $\lambda \neq 0$, $\mu \neq 0$ and $\lambda \neq \mu$

In accordance with annex **B.2.1** the feasible annihilating polynomial of the endomorphism associated to electromagnetic field **F**, is

$$\boxed{P_4(F) \equiv (F - \lambda I)(F + \lambda I)(F^2 + \mu^2 I);}$$

This is the einstenian case.

The next equations system fulfils the previous equation:

$$\begin{aligned} F(\overrightarrow{\mathbf{p}}) &= \lambda \overrightarrow{\mathbf{p}} \\ F(\overrightarrow{\mathbf{q}}) &= -\lambda \overrightarrow{\mathbf{q}} \\ F(\overrightarrow{\mathbf{Y}}) &= \mu \overrightarrow{\mathbf{Z}} \\ F(\overrightarrow{\mathbf{Z}}) &= -\mu \overrightarrow{\mathbf{Y}} \end{aligned} \tag{9}$$

Because of the skew-adjoint features of F, we have:

$$\begin{aligned} \overrightarrow{\mathbf{p}} F(\overrightarrow{\mathbf{p}}) &= \lambda \overrightarrow{\mathbf{p}}^2 = 0 \\ \overrightarrow{\mathbf{q}} F(\overrightarrow{\mathbf{q}}) &= -\lambda \overrightarrow{\mathbf{q}}^2 = 0 \\ \overrightarrow{\mathbf{Y}} F(\overrightarrow{\mathbf{Y}}) &= \mu \overrightarrow{\mathbf{Z}}.\overrightarrow{\mathbf{Y}} = 0 \\ \overrightarrow{\mathbf{Z}} F(\overrightarrow{\mathbf{Z}}) &= -\mu \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{Z}} = 0 \\ \overrightarrow{\mathbf{Z}} F(\overrightarrow{\mathbf{Y}}) &= \mu \overrightarrow{\mathbf{Z}}^2 \\ \overrightarrow{\mathbf{Y}} F(\overrightarrow{\mathbf{Z}}) &= -\mu \overrightarrow{\mathbf{Y}}^2 \\ \overrightarrow{\mathbf{Y}} F(\overrightarrow{\mathbf{Z}}) + \overrightarrow{\mathbf{Z}} F(\overrightarrow{\mathbf{Y}}) &= \mu(\overrightarrow{\mathbf{Z}}^2 - \overrightarrow{\mathbf{Y}}^2) = 0 \end{aligned} \tag{10}$$

It is clear that

$$\overrightarrow{\mathbf{p}}^2 = \overrightarrow{\mathbf{q}}^2 = 0$$

$$\overrightarrow{\mathbf{Y}}^2 = \overrightarrow{\mathbf{Z}}^2$$

Namely, $\overrightarrow{\mathbf{p}}$ and $\overrightarrow{\mathbf{q}}$ are null vectors.
It is not hard to see that

$$\begin{aligned}
\overrightarrow{\mathbf{Y}} F(\overrightarrow{\mathbf{p}}) &= \lambda \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{p}} \\
\overrightarrow{\mathbf{p}} F(\overrightarrow{\mathbf{Y}}) &= \mu \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{Z}} \\
\overrightarrow{\mathbf{Y}} F(\overrightarrow{\mathbf{q}}) &= \lambda \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{q}} \\
\overrightarrow{\mathbf{q}} F(\overrightarrow{\mathbf{Y}}) &= \mu \overrightarrow{\mathbf{q}}.\overrightarrow{\mathbf{Z}} \\
\overrightarrow{\mathbf{Z}} F(\overrightarrow{\mathbf{p}}) &= \lambda \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{Z}} \\
\overrightarrow{\mathbf{p}} F(\overrightarrow{\mathbf{Z}}) &= -\mu \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{Y}} \\
\overrightarrow{\mathbf{Z}} F(\overrightarrow{\mathbf{q}}) &= -\lambda \overrightarrow{\mathbf{Z}}.\overrightarrow{\mathbf{q}} \\
\overrightarrow{\mathbf{q}} F(\overrightarrow{\mathbf{Z}}) &= -\mu \overrightarrow{\mathbf{q}}.\overrightarrow{\mathbf{Y}}
\end{aligned} \tag{11}$$

From 11 it is also derived:

$$\begin{aligned}
\lambda \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{p}} &= -\mu \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{Z}} \\
\lambda \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{q}} &= \mu \overrightarrow{\mathbf{q}}.\overrightarrow{\mathbf{Z}} \\
\lambda \overrightarrow{\mathbf{Z}}.\overrightarrow{\mathbf{p}} &= \mu \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{Y}} \\
\lambda \overrightarrow{\mathbf{Z}}.\overrightarrow{\mathbf{q}} &= -\mu \overrightarrow{\mathbf{q}}.\overrightarrow{\mathbf{Y}}
\end{aligned} \tag{12}$$

And from 12 it is also derived:

$$\begin{aligned}
(\lambda^2 + \mu^2) \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{p}} &= 0 \\
(\lambda^2 + \mu^2) \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{q}} &= 0 \\
(\lambda^2 + \mu^2) \overrightarrow{\mathbf{Z}}.\overrightarrow{\mathbf{p}} &= 0 \\
(\lambda^2 + \mu^2) \overrightarrow{\mathbf{Z}}.\overrightarrow{\mathbf{q}} &= 0
\end{aligned} \tag{13}$$

As we are in the real field we have

$$\begin{aligned}
\overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{p}} &= 0\\
\overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{q}} &= 0\\
\overrightarrow{\mathbf{Z}}.\overrightarrow{\mathbf{p}} &= 0\\
\overrightarrow{\mathbf{Z}}.\overrightarrow{\mathbf{q}} &= 0
\end{aligned} \tag{14}$$

Fitting the scale of $\overrightarrow{\mathbf{Y}}$ and $\overrightarrow{\mathbf{Z}}$, we can infer

$$\begin{aligned}
\overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{q}} &= w\\
\overrightarrow{\mathbf{Z}}^2 = \overrightarrow{\mathbf{Y}}^2 &= 1
\end{aligned} \tag{15}$$

Therefore, in short we have

$$\overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{q}} = w; \quad \overrightarrow{\mathbf{Y}}^2 = \overrightarrow{\mathbf{Z}}^2 = 1; \quad \overrightarrow{\mathbf{p}}^2 = \overrightarrow{\mathbf{q}}^2 = 0$$

$$\mathbf{p}, \mathbf{q} \perp \mathbf{Y}, \mathbf{Z}; \quad \mathbf{Y} \perp \mathbf{Z}$$

.

D The time into the metric

Into the context of Annex **B.2.1**, we still have not studied in what term of the metric
$ds^2 = -dx^{0^2} + \sum_{i=1}^{i=3} dx^{i^2}$, the time shows up.

The einstenian theory of relativity arises from the fact that
$dx^0 = cdt$, therefore $ds^2 = -c^2dt^2 + \sum_{i=1}^{i=3} dx^{i^2}$.

In this context we have a null cone that splits the space-time in two parts: *timelike* and *spacelike*.

$$timelike \equiv ds^2 < 0$$

$$spacelike \equiv ds^2 > 0$$

and also the null cone:

$$lightlike \equiv ds^2 = 0$$

If instead of $dx^0 = cdt$, we have to analyze, for example, $dx^1 = cdt$; then, $ds^2 = +c^2dt^2 - (dx^0)^2 + \sum_{i=2}^{i=3} dx^{i^2}$

Here dx^0 is a spacelike coordinate, not a timelike coordinate. Then

$$timelike \equiv ds^2 > 0$$

$$spacelike \equiv ds^2 < 0$$

$$+c^2dt^2 - (dx^0)^2 + \sum_{i=2}^{i=3} dx^{i^2} < 0$$

it must be

$$+c^2dt^2 + \sum_{i=2}^{i=3} dx^{i^2} < (dx^0)^2$$

and also the null cone:

$$lightlike \equiv ds^2 = 0$$

In this case we are into an **spacelike hiperbolic, contrary to physical reality** (spacelike is to be eliptic).

In short, the time brings out in the negative term $-dx^{0^2}$ of the metric

$$ds^2 = -dx^{0^2} + \sum_{i=1}^{i=3} dx^{i^2}$$

Here, it is $dx^0 = cdt$

E Coclusions

We have made a new relativistic scheme only on the basis of a four-dimensional space, the skew-adjoint structure of the electromagnetic field, and a not defined metric, or rather, a metric with signature unknown for the moment.

In 1 we have an equation of degree 4, because of the four-dimensional space (for the moment has nothing to do with space-time). From the 5 degree, the solutions of algebraic equations cannot be expressed as algebraic function (that is, only with addition, subtraction, multiplication, division, and roots). For us, is a reason to deem the space is four-dimensional.

However,it is feasible an annihilating polynomial similar to 1, but of degree higher than 4. Then it is necessary to allow the factorization of this polynomial.

Otherwise, it is not possible to construct the lagrangian, in general.

The skew-adjoint structure of the electromagnetic field, implies the existence of a metric, although the signature is not necessarily to know in the beginning.

Summing up, we attain the next important outcome:

The electromagnetic field has his own space-time.

We dare o tell that any other basic field, has his own space-time. The physical phenomena derived of these fields act into his space-time.

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