Some concepts about electric field, magnetic field and invariants of electromagnetic tensor.

F. Sánchez M. *†

September 7, 2016

Abstract

In this article we report some results related to the electric and magnetic fields in connection with the electromagnetic field structure on which we contributed in the previous article "Structures of the Skew-adjoint Endomorphisms and Some Peculiarities of Electromagnetic Field" (see [3]).

Mainly we show the electric and magnetic field formulation in a reference frame in relative rest and his relation with electromagnetic field invariants λ and μ that we derived in the aforementioned article.

*Licenciate in Physical Sciences (Universidad Complutense de Madrid; Spain)

†High Statal Public Body of Systems and Technologies in Computers and Communications (Spain) ; High Statal Public Body of Meteorology (Spain), Professor of computer sciences in the INM (Instituto Nacional de Meteorología ; now retired) ; e-mail fs@relativityworkshop.com

1 Introduction

We start from the conclusions of the article "Structures of the Skew-adjoint Endomorphisms and Some Peculiarities of Electromagnetic Field". See http://relativityworkshop.com/paper3.pdf. In this article we are dealing with the electromagnetic tensor

$$\mathbf{F} = \lambda\overrightarrow{\mathbf{U}} \wedge \overrightarrow{\mathbf{X}} + \mu\overrightarrow{Y} \wedge \overrightarrow{\mathbf{Z}} \tag{1}$$

λ and μ are the invariants of the electromagnetic field that we find in the aforementioned article [1]. We search the connection among electromagnetic invariants λ, μ, and the electric field E and magnetic field H to have a thorough outlook of our theory of electromagnetic field.

The physical laws in macrophysics rest upon electromagnetic fields together the quantum phenomena. For this reason electromagnetic fields penetrate deeply into the physical world. On this basis we think it is necessary to work out the development of a relevant analysis on electromagnetic fields.

In Annex A we show without proving the main outcomes of the mentioned article "Structures of the Skew-adjoint Endomorphisms and Some Peculiarities of Electromagnetic Field" (see [3]), in order for gaining more comprehension reading this paper.

2 Notations, symbols and terminology

Vectors are symbolized with over right arrow.

Tensors stand for bold uppercase letters.

Endomorphisms stand for normal uppercase letters.

Subscripts are symbolized by lower case greek letters, saving λ and μ that are used to denote invariants. Also can be symbolized by latin alphabet letters .

The matrix of components of an endomorphism, tensor, etc.. can be shown closing inside parenthesis the symbol of this endomorphism , tensor, etc.. .

For example $(\mathbf{T})$ can stands for the matrix of components of $\mathbf{T}$. $(\mathbf{g}_{\alpha\beta})$ is a matrix whose elements are $\mathbf{g}_{\alpha\beta}$.

The two vectors scalar product $\overrightarrow{\mathbf{x}}$ and $\overrightarrow{\mathbf{y}}$ is symbolized by $\mathbf{G}(\overrightarrow{\mathbf{x}}, \overrightarrow{\mathbf{y}})$ where $\mathbf{G}$ is the metric tensor. Also is symbolized by $\overrightarrow{\mathbf{x}}.\overrightarrow{\mathbf{y}}$.

[1] λ and μ are invariants inferred from the annihilating polynomial of endomorphism associated with electromagnetic tensor

3 Basics

We work on the electromagnetic field on the basis of a lorentzian space L_4 with signature (-1,1,1,1) endowed with a metric $\mathbf{G}$ non degenerated, that is

$det(G) \neq 0$.

$\mathbf{F}$ is the electromagnetic tensor and F is the endomorphism associated with this tensor.

We rest upon an orthonormal relative rest reference frame $(\overrightarrow{\mathbf{U}}, \overrightarrow{\mathbf{X}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$ where

$$\overrightarrow{\mathbf{X}}^2 = \overrightarrow{\mathbf{Y}}^2 = \overrightarrow{\mathbf{Z}}^2 = 1$$
$$\overrightarrow{\mathbf{U}}^2 = -1$$

This reference frame is a reference frame in which we shall depict the electromagnetic field tensor 3 and the stress energy tensor 5 in relative rest respect an observer (see [3]), (see also Annex B about how is inferred this reference frame).

The matrix of components of the endomorphism associated with electromagnetic tensor (or the electromagnetic tensor mix components) is:

$$(F_\alpha{}^\beta) = \begin{pmatrix} 0 & \lambda & 0 & 0 \\ \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \\ 0 & 0 & -\mu & 0 \end{pmatrix}; \alpha, \beta = 0, 3 \tag{2}$$

The matrix of the electromagnetic tensor covariant components is:

$$(\mathbf{F}_{\alpha\beta}) = \begin{pmatrix} 0 & \lambda & 0 & 0 \\ -\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \\ 0 & 0 & -\mu & 0 \end{pmatrix}; \alpha, \beta = 0, 3 \tag{3}$$

The covariant dyadic form of electromagnetic tensor is:

$$\mathbf{F} = \lambda \overrightarrow{\mathbf{U}} \wedge \overrightarrow{\mathbf{X}} + \mu \overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}} \tag{4}$$

In the same reference frame, the matrix components of the endomorphism associated to stress-energy tensor is

$$(\mathbf{T}_\alpha{}^\beta) = \begin{pmatrix} -\chi^2 & 0 & 0 & 0 \\ 0 & -\chi^2 & 0 & 0 \\ 0 & 0 & \chi^2 & 0 \\ 0 & 0 & 0 & \chi^2 \end{pmatrix} \tag{5}$$

where $\chi^2 = \frac{1}{2}(\lambda^2 + \mu^2)$

4 Electric and magnetic fields formulation

It is necessary to analyse severally the relativistic formulation of electric and magnetic fields, $\overrightarrow{\mathbf{E}}$ and $\overrightarrow{\mathbf{H}}$.

$$\overrightarrow{\mathbf{E}} = \mathbf{F}.\overrightarrow{\mathbf{U}} \tag{6}$$

$$\overrightarrow{\mathbf{H}} = \mathbf{F}^{*}.\overrightarrow{\mathbf{U}} \tag{7}$$

Using components we have:

$$\mathbf{E}_{\alpha} = \mathbf{F}_{\alpha}{}^{\beta}\mathbf{U}_{\beta} \tag{8}$$

$$\mathbf{H}_{\alpha} = \eta^{\alpha\beta\gamma\delta}\mathbf{U}_{\beta}\mathbf{F}_{\gamma\delta} \tag{9}$$

where $\eta_{\alpha\beta\gamma\delta} = \sqrt{-g}\epsilon_{\alpha\beta\gamma\delta}$; $\eta^{\alpha\beta\gamma\delta} = \frac{1}{\sqrt{-g}}\epsilon_{\alpha\beta\gamma\delta}$ and where $g = det(G)$ and $\epsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol.

5 Relation among electric and magnetic fields and invariants

Taking into account 2 , 6 , 7, 8, and 9 :

$$\overrightarrow{\mathbf{E}} = \lambda\overrightarrow{\mathbf{X}}$$

that is

$$\lambda = |\overrightarrow{\mathbf{E}}|$$

$$\mathbf{H}_{\alpha} = \eta_{\alpha\beta\gamma\delta}\mathbf{U}^{\beta}\mathbf{F}^{\gamma\delta} = \mu\eta_{\alpha\beta\gamma\delta}\mathbf{U}^{\beta}\mathbf{Y}^{\gamma}\mathbf{Z}^{\delta}$$

$$\overrightarrow{\mathbf{H}}.\overrightarrow{\mathbf{X}} = \eta^{\alpha\beta\gamma\delta}\mathbf{X}_{\alpha}\mathbf{U}_{\beta}\mathbf{F}_{\gamma\delta} = \mu\eta^{\alpha\beta\gamma\delta}\mathbf{U}_{\alpha}\mathbf{X}_{\beta}\mathbf{Y}_{\gamma}\mathbf{Z}_{\delta}$$

and taking into regard

$$\eta^{\alpha\beta\gamma\delta}\mathbf{U}_{\alpha}\mathbf{X}_{\beta}\mathbf{Y}_{\gamma}\mathbf{Z}_{\delta} = \sqrt{-g}$$

it is not hard to see

$$\overrightarrow{\mathbf{H}} = \mu\overrightarrow{\mathbf{X}}$$

so

$$\mu = |\overrightarrow{\mathbf{H}}|$$

and thereby

$$\overrightarrow{\mathbf{H}}||\overrightarrow{\mathbf{E}}$$

Summing up we have:

$$\overrightarrow{\mathbf{E}} = \lambda\overrightarrow{\mathbf{X}}$$
$$\overrightarrow{\mathbf{H}} = \mu\overrightarrow{\mathbf{X}}$$
$$\lambda = |\overrightarrow{\mathbf{E}}|$$
$$\mu = |\overrightarrow{\mathbf{H}}|$$

Therefore

$$I_1 = \mu^2 - \lambda^2 = |\overrightarrow{\mathbf{H}}|^2 - |\overrightarrow{\mathbf{E}}|^2$$
$$I_2 = \mu\lambda = \overrightarrow{\mathbf{H}}.\overrightarrow{\mathbf{E}} = |\overrightarrow{\mathbf{H}}|.|\overrightarrow{\mathbf{E}}|$$

where I_1 and I_2 are the classic invariants of the electromagnetic fields referred to a reference frame adapted to the base $(\overrightarrow{\mathbf{U}}, \overrightarrow{\mathbf{X}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$

It is worthwhile to point out the impact that a change of base has on fields $\overrightarrow{\mathbf{E}}$ and $\overrightarrow{\mathbf{H}}$.

According with 6 and 7 $\overrightarrow{\mathbf{E}}$ and $\overrightarrow{\mathbf{H}}$ depend on $\overrightarrow{\mathbf{U}}$ that is a component of the reference frame $(\overrightarrow{\mathbf{U}}, \overrightarrow{\mathbf{X}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$.

In general $\overrightarrow{\mathbf{H}}$ and $\overrightarrow{\mathbf{E}}$ are not invariants (since they depend on $\overrightarrow{\mathbf{U}}$ as we have aforementioned). However $|\overrightarrow{\mathbf{H}}|^2 - |\overrightarrow{\mathbf{E}}|^2$ and $\overrightarrow{\mathbf{H}}.\overrightarrow{\mathbf{E}}$ are invariant in a lorentz transformation [2].

[2] It is proved in many texts of general relativity. In the time being we do not attempt to prove it. We will prove it when we deal with lorentz transformation in afterward articles.

ANNEXES

A Geometrical structure of electromagnetic field

A.1 Basics

In the theory of relativity the physical magnitudes are tensorial. Every two order tensor can be mix (besides covariant and contravariant) tensor. Mix tensor is equivalent to the associated endomorphism. We single out the metric tensor; his associated endomorphism is *the identity endomorphism.*

Here we go into the skew-adjoint endomorphism in order to study closely the electromagnetic field. Therefor we develop a study and analysis on the base of the endomorphism associated with the skew-adjoint tensor namely to the electromagnetic tensor (F symbolizes the endomorphism associated to electromagnetic field) . On this base we can use the annihilating polynomial of F to classify, to structure and to give form to electromagnetic field.

A.2 Annihilating polynomials in a lorentzian space-time.

Following a straightforward way we begin with the analysis of invariant subspaces in a lorentzian space L_4 constructing them out of the usual basis of annihilating polynomials into L_4 or his subspaces. The invariant subspaces are lorentzian, euclídean or null. Thereby firstly my purpose is the analysis of annihilating polynomials regarding annihilating minimal polynomials where relevant.

A.2.1 Study of annihilating polynomial of grade 4, of a skew-adjoint endomorphism A in a minkowskian space L_4.

The most general case of annihilating polynomial in our context of lorentzian space L_4 is:

$$P(F) = F^4 + a_3F^3 + a_2F^2 + a_1F + a_0I$$

We have

$$\forall\overrightarrow{\mathbf{X}};\overrightarrow{\mathbf{X}} \in L_4; P(F)\overrightarrow{\mathbf{X}} = 0$$

It is easily verified

$$\forall\overrightarrow{\mathbf{X}},\overrightarrow{\mathbf{Y}} \in L_4 \quad ; \quad g(\overrightarrow{\mathbf{X}}, P(F)\overrightarrow{\mathbf{Y}}) = 0 \qquad g(\overrightarrow{\mathbf{Y}}, P(F)\overrightarrow{\mathbf{X}}) = 0 \tag{10}$$

Adding and subtracting equations 10 , and taking into account that F^2 and F^4 are self-adjoint endomorphisms, and that F and F^3 are skew-adjoint endomorphisms we have

$$\forall \overrightarrow{\mathbf{X}}, \overrightarrow{\mathbf{Y}} \in L_4 \ ; g(\overrightarrow{\mathbf{X}}, (F^4 + a_2F^2 + a_0I)\overrightarrow{\mathbf{Y}}) = 0$$
$$; \ g(\overrightarrow{\mathbf{X}}, (a_3F^3 + a_1F)\overrightarrow{\mathbf{Y}}) = 0$$

One can easily verify

$$\begin{aligned} P_4(A) &\equiv F^4 + a_2F^2 + a_0I = (0) \\ P_3(A) &\equiv (F)(a_3F^2 + a_1) = (0) \end{aligned} \tag{11}$$

In this article we only will be concerned with the first equation of 11.

So the annihilating polynomial is:

$$P_4(F) \equiv F^4 + a_2F^2 + a_0I = (0)$$

[3]

Summing up equation $P_4(F) \equiv F^4 + a_2F^2 + a_0I = (0)$, depicts annihilating polynomial on L_4 that concern us.

A.2.2 Factorization of Annihilating polynomial on L_4.

According to the foregoing sections, the goal of this article is the examination of skew-adjoint endomorphism to investigate afterward more closely electromagnetic field.

Here we are only concerned with the factorized polynomial:

$$P_4(F) \equiv (F^2 + \epsilon\lambda^2 I)(F^2 + \eta\mu^2 I) \tag{12}$$

$\epsilon = \pm 1$; $\eta \pm 1$

As it is easily checked

$$a_2 = \epsilon\lambda^2 + \eta\mu^2 \tag{13}$$

$$a_0 = \epsilon\eta\lambda^2\mu^2 \tag{14}$$

Further along, in other sections, we shall study the classification of annihilating polynomial $P_4(F) \equiv F^4 + a_2F^2 + a_0I = (0)$ on the basis of λ and μ.

[3] However that does not mean we have to rule out the second case $P_3(A) \equiv (F)(a_3F^2 + a_1) = (0)$. It is worthwhile to examine it aside outside this article. However henceforth we confine only to the first equation of 11.

A.2.3 Orthogonality relation between $\ker(A^2+\epsilon\lambda^2 I)$ and $\ker(A^2+\eta\mu^2 I)$

It is proved (see [3])

$$\lambda \neq 0 \quad \text{or} \quad \mu \neq 0 \quad \text{and} \quad \epsilon\lambda^2 \neq \eta\mu^2 \tag{15}$$

involves orthogonality between $\ker(A^2+\epsilon\lambda^2 I)$ and $ker(A^2+\eta\mu^2 I)$ with zero intersection that is $L_4 \equiv ker(A^2+\epsilon\lambda^2 I) \overset{\perp}{\oplus} ker(A^2+\mu^2 I)$.

We call regular cases to the endomorphisms that verify 15.

As complement it is worthwhile to prove : If $\ker(F^2+\epsilon\lambda^2 I)\cap\ker(F^2+\mu\lambda^2 I) \neq \emptyset$ then

$$\lambda = 0 \quad \text{and} \quad \mu = 0 \quad \text{or} \quad \epsilon\lambda^2 = \eta\mu^2 \tag{16}$$

In the following sections we analyze invariants subspaces of A, together with their minimal polynomials when relevant.

Notice if $\epsilon \neq \eta$ it is sufficient $\lambda \neq 0$ or $\mu \neq 0$ to verify 15 .

A.2.4 Annihilating polynomials $(F^2+\epsilon\lambda^2 I)$, $(F^2+\eta\mu^2 I)$ and their invariant subspaces structure

.

In this stage we are in the context 15. So we begin with the case we call *regular field or pure field* [4].

We only analyze cases in the context of regular case. We select the case $\epsilon = 1$ and $\eta = -1$ from which we shall constitute all other cases in the framework of *pure or regular field*.

Summing up

$$L_2 \equiv ker(F^2 - \lambda^2 I)$$

L_2 **has a base** $(\overrightarrow{\mathbf{p}}, \overrightarrow{\mathbf{q}})$ **, where** $\overrightarrow{\mathbf{p}}$ **and** $\overrightarrow{\mathbf{q}}$ **are null eigenvectors of** L_2 **with eigenvalues** $+\lambda$ and $-\lambda$

$$E_2 \equiv ker(F^2 + \mu^2 I)$$

E_2 **is not decomposable**.

$$L_4 = L_2 \overset{\perp}{\oplus} E_2$$

Henceforth we shall use these notations L_2 and E_2 for $ker(F^2-\lambda^2 I)$ and $ker(F^2+\mu^2 I)$

[4] *pure field or regular field* are those that fulfil the conditions itemized in previous section, that is $\lambda \neq 0$ or $\mu \neq 0$, and $\lambda^2 \neq \mu^2$

This case is called *pure field or regular case.*

It is not hard to prove that all other regular cases either are composed of cases 1) and 2) or they are incongruous in our context.

A.3 Invariants λ and μ

.

Therefore the minimal polynomial on L_4 that concerns us (in the regular case) is

$$P(F) = F^4 + a_2F^2 + a_0I \equiv (F^2 - \lambda^2 I)(F^2 + \mu^2 I)$$

λ and μ are functions of coefficients of $P(A)$ (therefore invariants).

Then we have the invariants

$$\begin{aligned} I_1 &= \mu^2 - \lambda^2 = a_2 \\ I_2 &= -\mu^2\lambda^2 = a_0 \end{aligned} \tag{17}$$

Further along, it will be single out that the invariants λ and μ (or I_1 and I_2) play a relevant role in electromagnetic field.

A.4 Tensorial representation of skew-adjoint endomorphisms

In this section we shall illustrate the tensorial applications going into details, namely developing the tensor components in the two reference frames we show in the following.

In an endomorphism F *associated* with a tensor $\mathbf{F}$, the components of endomorphism F are the mix components of a two-order tensor $\mathbf{F}$ with the same basis in they both (F and $\mathbf{F}$).

As a matter of fact, the tensorial mix components ${\mathbf{F}_\alpha}^\beta$ and the components of the associated endomorphism are the same (in connection with the same base).

The tensorial representations involves a metric tensor $\mathbf{G}$, in which covariants components are $\mathbf{G}_{\alpha\beta}$ and contravariant components are $\mathbf{G}^{\alpha\beta}$.

Therefore we have

$$(\mathbf{F}^{\alpha\beta}) = (\mathbf{G}^{\alpha\lambda}{\mathbf{F}_\lambda}^\beta)$$

A.4.1 Tensorial representation applied to regular case (pure field)

We are working on $L_2 \overset{\perp}{\oplus} E_2$

$(\overrightarrow{\mathbf{p}}, \overrightarrow{\mathbf{q}})$ is a base on L_2, and $(\overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$ is an ortonormal base on E_2.

In this article we develop the components of tensor $\mathbf{F}$ in two reference frames: the mentioned $(\overrightarrow{\mathbf{p}}, \overrightarrow{\mathbf{q}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$ and the $(\overrightarrow{\mathbf{U}}, \overrightarrow{\mathbf{X}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$ where

$$\overrightarrow{\mathbf{p}} = a(\overrightarrow{\mathbf{X}} + \varepsilon\overrightarrow{\mathbf{U}})$$

$$\overrightarrow{\mathbf{q}} = b(\overrightarrow{\mathbf{X}} - \varepsilon\overrightarrow{\mathbf{U}}); \varepsilon = \pm 1$$

(see Annex B)

A.4.2 Reference frame $(\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{Y}}, \vec{\mathbf{Z}})$

In L_2 we rest upon the existence of the scalar product $w = \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{q}}$ to define the metric on L_2.
In the reference frame $(\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{Y}}, \vec{\mathbf{Z}})$ the metric in L_4 in covariant components is

$$(G_{\alpha\beta}) = \begin{pmatrix} 0 & w & 0 & 0 \\ w & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \alpha, \beta = 0, 3$$

and in contravariant components

$$(G^{\alpha\beta}) = \begin{pmatrix} 0 & \frac{1}{w} & 0 & 0 \\ \frac{1}{w} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \alpha, \beta = 0, 3$$

Agree on foregoing propositions we have $\overrightarrow{\mathbf{p}}, \overrightarrow{\mathbf{q}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}}$ verifies

$$\overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{q}} = w; \quad \overrightarrow{\mathbf{Y}}^2 = \overrightarrow{\mathbf{Z}}^2 = 1; \quad \overrightarrow{\mathbf{p}}^2 = \overrightarrow{\mathbf{q}}^2 = 0$$

$$\mathbf{p}, \mathbf{q} \perp \mathbf{Y}, \mathbf{Z}; \quad \mathbf{Y} \perp \mathbf{Z}$$

.

The physical covariant dyadic representation of the tensor $\mathbf{F}$ is

$$\mathbf{F} = \lambda w \overrightarrow{\mathbf{p}} \wedge \overrightarrow{\mathbf{q}} + \mu \overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}} \tag{18}$$

The electromagnetic field we deem here is similar to the found in [2] if $w = 1$.

The dyadic representation of the tensor in contravariant components is

$$\mathbf{F} = -\lambda\frac{1}{w}\overrightarrow{\mathbf{p}} \wedge \overrightarrow{\mathbf{q}} + \mu\overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}}; \tag{19}$$

It is interesting to point out that w appears only into the context of covariant and contravariant components.

A.4.3 Reference frame: $(\vec{\mathbf{U}}, \vec{\mathbf{X}}, \vec{\mathbf{Y}}, \vec{\mathbf{Z}})$

Herein we are dealing with a reference frame associated to observer and with the invariant subspaces of the endomorphism F (see Annex B).

This reference frame is $(\overrightarrow{\mathbf{U}}, \overrightarrow{\mathbf{X}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$ and constitute an orthonormal base.

In this reference frame the metric tensor is

$$(\mathbf{G}^{\alpha\beta}) = (\mathbf{G}_{\alpha\beta}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \alpha, \beta = 0, 3$$

In the dyadic depiction of the tensor (in this case only for mix tensor field components) we have

$$\mathbf{F} = \lambda(\overrightarrow{\mathbf{U}} \otimes \overrightarrow{\mathbf{X}} + \overrightarrow{\mathbf{X}} \otimes \overrightarrow{\mathbf{U}}) + \mu\overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}} \tag{20}$$

This result agree to [5]

In the dyadic depiction of the tensor (in this case only for covariant tensor field components) we have

$$\mathbf{F} = \lambda\overrightarrow{\mathbf{U}} \wedge \overrightarrow{\mathbf{X}} + \mu\overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}} \tag{21}$$

These results are similar to [6].

A.5 Electromagnetic tensor

The types of electromagnetic field we show in this article agree on 16. We call *pure fields* to these electromagnetic field. Pure electromagnetic field is the case that concern us in this article.

On the base $(\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{Y}}, \vec{\mathbf{Z}})$ the electromagnetic tensor field with covariant components 18 in the dyadic context is :

$\mathbf{F} = w\lambda\overrightarrow{\mathbf{p}} \wedge \overrightarrow{\mathbf{q}} + \mu\overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}}$

and with contravariant components 19 in the dyadic context is :

$\mathbf{F} = -\lambda\frac{1}{w}\overrightarrow{\mathbf{p}} \wedge \overrightarrow{\mathbf{q}} + \mu\overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}}$

On the base $(\vec{\mathbf{U}}, \vec{\mathbf{X}}, \vec{\mathbf{Y}}, \vec{\mathbf{Z}})$ the electromagnetic field mix tensor is
$\mathbf{F} = \lambda(\overrightarrow{\mathbf{U}} \otimes \overrightarrow{\mathbf{X}} + \overrightarrow{\mathbf{X}} \otimes \overrightarrow{\mathbf{U}}) + \mu\overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}}$
and in covariant context is
$\mathbf{F} = \lambda\overrightarrow{\mathbf{U}} \wedge \overrightarrow{\mathbf{X}} + \mu\overrightarrow{\mathbf{Y}} \wedge \overrightarrow{\mathbf{Z}}$
In these cases it is suitable point out $\lambda \neq 0$ or $\mu \neq 0$.

B Concepts about referential frame at rest respect an observer.

B.1 Definition of reference frame at rest respect an observer.

We define a reference frame at relative rest related to an observer, as reference frame such that the matrix $(\mathbf{G_r})$ of the metric tensor covariant or contravariant components in this reference frame is:

$$(\mathbf{G_r}) = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Thereby the vectorial base of the reference frame at relative rest respect an observer must be orthonormal.

B.2 Passage from the pseudo-orthonormal base to an orthonormal base at rest.

That is from the pseudo-orthonormal base $(\overrightarrow{\mathbf{p}}, \overrightarrow{\mathbf{q}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$ to the orthonormal base $(\overrightarrow{\mathbf{U}}, \overrightarrow{\mathbf{X}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$

As we saw earlier in a pseudo-orthonormal base we have:

$$\overrightarrow{\mathbf{p}}^2 = \overrightarrow{\mathbf{q}}^2 = 0; \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{q}} = w; \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{Z}} = \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{Y}} = \overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{Z}} = \overrightarrow{\mathbf{q}}.\overrightarrow{\mathbf{Y}} = \overrightarrow{\mathbf{q}}.\overrightarrow{\mathbf{Z}} = 0$$

$$\overrightarrow{\mathbf{Y}}^2 = \overrightarrow{\mathbf{Z}}^2 = 1$$

In the orthonormal basis $(\overrightarrow{\mathbf{U}}, \overrightarrow{\mathbf{X}}, \overrightarrow{\mathbf{Y}}, \overrightarrow{\mathbf{Z}})$ we have

$$\overrightarrow{\mathbf{U}}^2 = -1; \overrightarrow{\mathbf{X}}^2 = \overrightarrow{\mathbf{Y}}^2 = \overrightarrow{\mathbf{Z}}^2 = 1$$

$$\overrightarrow{\mathbf{U}}.\overrightarrow{\mathbf{X}} = \overrightarrow{\mathbf{U}}.\overrightarrow{\mathbf{Y}} = \overrightarrow{\mathbf{U}}.\overrightarrow{\mathbf{Z}} = \overrightarrow{\mathbf{X}}.\overrightarrow{\mathbf{Y}} = \overrightarrow{\mathbf{X}}.\overrightarrow{\mathbf{Z}} = \overrightarrow{\mathbf{Y}}.\overrightarrow{\mathbf{Z}} = 0$$

The passage equations are:

$$\overrightarrow{\mathbf{p}} = a_p\overrightarrow{\mathbf{X}} + b_p\overrightarrow{\mathbf{U}}$$

$$\overrightarrow{\mathbf{q}} = a_q \overrightarrow{\mathbf{X}} + b_q \overrightarrow{\mathbf{U}}$$

$\overrightarrow{\mathbf{Y}}$ and $\overrightarrow{\mathbf{Z}}$ remain the same.

It is easily checked that:

$$a_p = \varepsilon b_p = a$$

$$a_q = \eta b_q = b$$

It must be $\eta = -\varepsilon$. Then the transition equations become:

$$\mathbf{p} = a(\varepsilon \overrightarrow{\mathbf{U}} + \overrightarrow{\mathbf{X}})$$

$$\mathbf{q} = b(-\varepsilon \overrightarrow{\mathbf{U}} + \overrightarrow{\mathbf{X}})$$

Here we have $\overrightarrow{\mathbf{p}}.\overrightarrow{\mathbf{q}} = 2ab = w$.

To keep the orientation toward the future must be $a > 0$, $b < 0$, $\varepsilon = +1$, thereby $w = 2ab < 0$

Then

$$\mathbf{p} = a(\overrightarrow{\mathbf{U}} + \overrightarrow{\mathbf{X}})$$
$$\mathbf{q} = b(-\overrightarrow{\mathbf{U}} + \overrightarrow{\mathbf{X}})$$

References

[1] **H. Stephani ;General Relativity (Cambridge University Press 1982); Special properties of source free fields; pag 72 ; interesting to study electromagnetism in vacuo.**

[2] **B. Coll;Concepts for a theory of the electromagnetic field (2003) gr-qc/0302056v1. Throw electromagnetic field in a new light.**

[3] **F.Sánchez; Structures of the Skew-adjoint Endomorphisms and Some Peculiarities of Electromagnetic Field.(2014)**

[4] **Electrovacuum solutions; wikipedia; references Stephani and others ; Exact solutuion of Einstein equations**

[5] **Gregory L. Naber; The geometry of Minkwski spacetime ;(Springer Verlag 1982) ;Section 2.4**

[6] **C. W. Misner and others; Gravitation ; W.H. Freeman and Company 1970 ; pag 122**