Insights into the Theory of Relativity.
Part I.
Critical Approach.
Basic Principles and Starting Points.

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Abstract

This scientific article develops the theory of relativity regardless of the principles "constancy of light speed", "homogeneity and isotropy of space", and "timing of clocks" in a minkowskian space-time on the basis of electromagnetic fields and reference frames features. In this article we do not think into the invariance of Maxwell equations. **It is proved that in this context orthogonal transformation preserves the skew-adjoint property of electromagnetic field**. Thereby **it is derived the Lorentz transformations** and (in part II) the Lorentz boost.

Some possible appealing generalizations arise from the hints that appear in the analysis of this work.

†The theory of relativity is rediscovered from new standpoints and principles.

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1 Introduction

In this paper I intend to develop new insights in the theory of relativity. I think I throw some new concepts in a new light. For the moment, the main outcome I get is the deduction of Lorentz transformation regardless of some principles as the homogeneity and isotropy of space, rigidity of bodies, Maxwell equations and constancy of velocity of light. Partially I agree to some other principles as the relativity principle. This paper is reported in two parts.

In the first part of this paper we develop basic starting points fundamental to establish some propositions and necessary principles. We rediscover the orthogonal transformation between two reference frames namely the Lorentz transformation even if in a more generalized way. The second part rests upon them.

In the second part of the paper we rediscover Lorentz boost transformation on the basis of the principles and outcomes explained in the first part [1].

From the beginning we are working on the basis of two observers that have their own space-time with the same features (namely dimension, the same signature, etc..) everyone. These space-times are real vectorial lorentzian space-times with signature (-1,1,1,1) endowed with a metric non degenerated.

In this paper it is relevant to single out the known importance of electromagnetic field in the theory of relativity. Poincaré [2] set up and gave a great relevance to the role that equations of Maxwell play in the theory of relativity. The electromagnetic field is so attached to the theory of relativity that the theory of relativity is to be constructed out of the electromagnetic field structure [2]. Actually electromagnetic fields penetrate deeply into the mechanics of the physical world.

Along this paper we are reducing to the cases that are close to the known relativity theory, leaving aside other options that are interestingly suitable to develop new generalizations. For the moment they are beyond the scope of this paper. I think it is worthwhile to tackle them aside.

2 Introduction to the part I. Basics.

Before explaining the new insights we intend to develop in this paper , we deem necessary to highlight the following basic propositions: [3]:

1.-As we pointed out before, **we are working on the basis of a minkowskian space-time equivalent to a lorentzian vectorial space with signature (-1,1,1,1) on a real field.**

In this context, for any observer, we can define a metric G which involves the definition of interval of universe $ds^2 = -(dx^0)^2+(dx^1)^2+(dx^2)^2+(dx^3)^2$ at an inertial rest vectorial reference frame. We agree to (dx^0) involves $dx^0 = cdt$ where t is time measured by the observer and c is a constant. Later along we shall prove that c is the velocity of the light.

The intervals are measured in a reference frame [4].

[1]This is really important because from my standpoint the theory of relativity of Einstein is based in the development of the physical interpretation of Lorentz transformation.

[2] The theory of relativity of Einstein also takes into account other principles and postulates for example relativity principle, isotropy and homogeneity of space, the timing of clocks, and so on.

[3]For convenience, some times we manage without the parenthesis in order to handle symbols in a more easy way. See *Notations, symbols and terminology* in last pages of the paper.

[4] It is worthwhile to see the reference frame as three measuring rods and a watch likewise in the early theory of relativity. It is a way really simple to see a reference frame. However it helps to understand what a physical reference is, since it is very intuitive.

2.-We agree with the relativity principle regarding to the **tensorial nature of physical magnitudes in the minkowskian space-time** [5].

Along this paper we will only work on the *associated endomorphism* to a tensor unless other wise specified [6].

3.-In the **frame of the minkowskian space-time described in 1.-** it is proved that **electromagnetic field is an antisymmetric second order tensor** F. In this frame Maxwell equations in vacuum are:

$d_{ext}F = 0$; $d^{*}_{ext}F = 0$ ([7]).

For the moment we work regardless of the Lorentz transformation (even if we shall derive it after along) and Maxwell equations. We only take into account **the electromagnetic components of his antisymmetric tensor, in the form of his associated skew-adjoint endomorphism** .

4.-The electromagnetic interactions are to a large extent the interactions existing in the macrocosm. Therefore electromagnetic fields play an universal role in the interaction and in the transference of information in the macrocosm.

Usually electromagnetic interactions involve fields, waves and radiation. Generally they concern the movements and transformations of reference frames as well.

The signals among observers actually are a kind of waves or radiation (in the basic theory these do not affect to the reference frame movement). For these reasons we deem necessary to keep (and only abiding by transformations among reference frames) the structure of electromagnetic tensor field F (that is his skew-adjoint associated endomorphism characteristic). It must be invariant in the transformation among reference frames and their movements. Therefore in short **the skew-adjoint nature of F is invariant**. The electromagnetic field must keep its structure for the reference frames of observers. Further along it will be proved that the orthogonal transformations (Lorentz transformations) preserve this structure.

Summing up, the starting points in the theory explained in this paper are:

1.-We are working on the basis of a minkowskian space-time equivalent to a lorentzian vectorial space with signature (-1,1,1,1) on a real field.

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[5]The measurements and magnitudes taken by an observer are real numbers or variables arranged in a matrix manner.

[6] A tensor has associated what we call an *associated endomorphism*. This endomorphism is nothing but the same tensor in the mixed form.

[7] d_{ext} is the exterior differential and $*$ is the Hodge operator (or star operator).

2.-Physical magnitudes have tensor nature in the minkowskian space-time.

3.-Electromagnetic field is an antisymmetric second order tensor into the frame of the minkowskian space-time described in 1.-

4.-The skew-adjoint characteristic of the electromagnetic field has to be invariant.

2.1 Principle of coupling of observers.

Along this paper we are dealing with two observers O_1 and O_2 that observe the same event F_0. They both make measurements of their observations of a physical event F_0. **Everyone has his own space-time**, that is O_1 has his space-time $\mathbb{L}_{O_1}$ and O_2 has his space-time $\mathbb{L}_{O_2}$. The observer O_1 has his reference frame with his base B_1.The observer O_2 has his reference frame with his base B_2. F_1 and F_2 are the mixed tensors components of tensor measurements gotten by observers O_1 and O_2 (are the matrices of components of the associated endomorphism).

It is set up the next principle or **Principle coupling of observers** :

There are two bases B_1 and B_2 of observers O_1 and O_2 in such a way that measurements respect these bases are equal.

Therefore we have the next relation among matrices (managing without parenthesis of the matrices for convenience)

$$B_1F_1B_1^{-1} = B_2F_2B_2^{-1}$$

that is

$$F_1 = R_1F_2R_1^{-1} \qquad R_1 = B_1^{-1}B_2$$
$$F_2 = R_2F_1R_2^{-1} \qquad R_2 = B_2^{-1}B_1$$

For convenience we make

$$R_1 = R^{-1}$$

$$R_2 = R$$

$$F_2 = RF_1R^{-1} \qquad R = B_2^{-1}B_1 \tag{1}$$

The (R transformation is an homomorphism in the context of coordinates transformation ; see [6]).

Thereby the $\mathbf{F_1}$ **and** $\mathbf{F_2}$ **matrices are similar** [8] .

Actually R means the relation between the measurements of the event F_0 gotten by observers O_1 and O_2.

2.2 Coordinates transformation

We use the next matrix notation for the components of a vector $\overrightarrow{\mathbf{X}}$

$$(\overrightarrow{\mathbf{X}}) = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$(\overrightarrow{\mathbf{X}})^t = \begin{pmatrix} x^0 & x^1 & x^2 & x^3 \end{pmatrix}$$

Then let be $(\overrightarrow{\mathbf{X_{O_1}}})$ and $(\overrightarrow{\mathbf{Y_{O_1}}})$ matrices coordinates measured by observer O_1 and $(\overrightarrow{\mathbf{X_{O_2}}})$ and $(\overrightarrow{\mathbf{Y_{O_2}}})$ matrices coordinates measured by observer O_2, both of the event observed F_0 [9].

For observers O_1 and O_2 we have the next coordinate transformation (see ANNEX A):

$$(\overrightarrow{\mathbf{X_{O_1}}})^t = (\overrightarrow{\mathbf{X_{O_2}}})^t R$$

$$(\overrightarrow{\mathbf{Y_{O_1}}}) = R^t (\overrightarrow{\mathbf{Y_{O_2}}})$$

3 Some basic features of orthogonal homomorphisms.

Before starting the development of the analysis of orthogonal transformations focused toward the analysis of relativity theory (Lorentz transformations) we deem fit to go into details related with the cited orthogonal transformations and also with skew-adjoint endomorphisms F_1 and F_2. A transformation (namely homomorphism) R transforms endomorphisms F_i; $i = 1, 2$ as follows:

$$F_2 \rightarrow R F_1 R^{-1}$$

(R transformation is in the context of coordinates transformation).

[8] Actually F_1 and F_2 are the associated endomorphisms to electromagnetic tensors in view of the above introduction.

[9] **The** R **transformation also concerns the vector components of** $\overrightarrow{\mathbf{X}}$**.**

We are dealing with the homomorphism R that is acting between the space-time $\mathbb{L}_{O_1}$ of observer O_1 and the space-time $\mathbb{L}_{O_2}$ of observer O_2. We start from the next orthogonal homomorphism definition:

$$\forall \overrightarrow{\mathbf{X_{O_1}}}; \overrightarrow{\mathbf{Y_{O_1}}} \in \mathbb{L}_1; \forall \overrightarrow{\mathbf{X_{O_2}}}; \overrightarrow{\mathbf{Y_{O_2}}} \in \mathbb{L}_2;$$

we have

$$G_1(\overrightarrow{\mathbf{X_{O_1}}}, \overrightarrow{\mathbf{Y_{O_1}}}) = G_2(\overrightarrow{\mathbf{X_{O_2}}}, \overrightarrow{\mathbf{Y_{O_2}}})$$

where G_1 is the metric tensor in space-time $\mathbb{L}_1$ and G_2 is the metric tensor in space-time $\mathbb{L}_2$.

In a first stage we deem necessary to work out invariant relations between transformations namely homomorphisms [10] and adjoint operations. The matrix components of adjoint endomorphism $F_i^\sharp$ of F_i $(i = 1, 2)$ verify [11] :

$$(F_{(i)}^\sharp) = (G_i)(F_{(i)}^t(G_i^{-1}) \quad i = 1, 2$$

For convenience we manage without the matrices parenthesis; then we write down:

$$F_{(i)}^\sharp = G_i F_{(i)}^t G_i^{-1} \quad i = 1, 2$$

3.1 Orthogonal homomorphism and adjoint operator.

We prove that if

$$F_2^\sharp = R F_1^\sharp R^{-1}$$

that is

$$(R F_1 R^{-1})^\sharp = R F_1^\sharp R^{-1}$$

then R is an orthogonal transformation.

Of course:

$$(R F_1 R^{-1})^\sharp = G_2 (R F_1 R^{-1})^t G_2^{-1} = G_2 R^{-1^t} F_1^t R^t G_2^{-1} \tag{2}$$

acting into $\mathbb{L}_{\mathbf{O_2}}$.

On the other hand

$$R F_1^\sharp R^{-1} = R G_1 F_1^t G_1^{-1} R^{-1} \tag{3}$$

acting into $\mathbb{L}_{\mathbf{O_1}}$.

Hence from 2 and 3 it is derived

$$G_2 R^{t^{-1}} = R G_1 \tag{4}$$

[10] We deem that coordinates transformations has only to do with transformations of components of vectors and tensors regardless of vectors base.

[11] For a more detailed accounting about definition of G-adjoint endomorphism see [6]

therefore from 4 it is inferred

$$\boxed{G_2 = RG_1R^t} \tag{5}$$

As we saw earlier:

$$(\mathbf{X}^t_{\mathbf{O_1}}) = (\mathbf{X}^t_{\mathbf{O_2}})R$$

$$(\mathbf{Y}_{\mathbf{O_1}}) = R^t(\mathbf{Y}_{\mathbf{O_2}})$$

Therefore we have

$$\mathbf{X}^t_{\mathbf{O_1}} G_1 \mathbf{Y}_{\mathbf{O_1}} = \mathbf{X}^t_{\mathbf{O_2}} RG_1R^t \mathbf{Y}_{\mathbf{O_2}} = \mathbf{X}^t_{\mathbf{O_2}} G_2 \mathbf{Y}_{\mathbf{O_2}}$$

that is

$$\mathbf{X}^t_{\mathbf{O_1}} G_1 \mathbf{Y}_{\mathbf{O_1}} = \mathbf{X}^t_{\mathbf{O_2}} G_2 \mathbf{Y}_{\mathbf{O_2}}$$

therefore

$$\boxed{G_2(\mathbf{X}_{\mathbf{O_2}}, \mathbf{Y}_{\mathbf{O_2}}) = G_1(\mathbf{X}_{\mathbf{O_1}}, \mathbf{Y}_{\mathbf{O_1}})}$$

In conclusion R **is orthogonal in the sense we before mentioned**.

Inversely it is easily proved that an orthogonal coordinate transformation R verifies

$$(RF_1R^{-1})^\sharp = RF_1^\sharp R^{-1}$$

3.1.1 Orthogonal transformations R preserves the skew-adjoint structure of the endomorphisms F_i ; $i = 1, 2$.

We highlight : **An orthogonal transformation that transforms a skew-adjoint endomorphism into another skew-adjoint endomorphism must be orthogonal.**

In fact:

We have

$$F_2^\sharp = -F_2 \qquad F_1^\sharp = -F_1$$

Then

$$F_2^\sharp = (RF_1R^{-1})^\sharp = -F_2 = -RF_1R^{-1} = R(-F_1)R^{-1} = RF_1^\sharp R^{-1}$$

what lead us to

$$F_2^\sharp = RF_1^\sharp R^{-1}$$

In accordance with our foregoing proposition R is to be orthogonal. These propositions are thoroughly applicable to electromagnetic fields.

Actually, thereby R **becomes the Lorentz transformation**.

3.1.2 Reduction to classical theory of relativity.

Reducing us to the limit of classical theory of relativity it is $G_1 = G_2$ since specifically we are working in the frame of inertial reference frames.

Into this context, according with 5, and writing down $G = G_1 = G_2$ it is clear that G verifies

$$G = RGR^t \tag{6}$$

3.2 The transporter principle.

Observers O_1 and O_2 can be related in such a way that the reference frame of O_2 can be carried to the reference frame of O_1 [12] . The inertial referential frames are embedded in an affine space-time context (in a similar manner to the basic relativity theory).

Anyway for reason of the above mentioned , we can boil down $\mathbb{L}_{O_2}$ to $\mathbb{L}_{O_1}$. Within this context **the homomorphism R $\mathbb{L}_{O_1} \to \mathbb{L}_{O_2}$ will be treated as an endomorphism into $\mathbb{L}_{O_1}$ in the following sections and subsections.**

In this way we shall study the features of the structure of the above mentioned orthogonal homomorphism R, as an orthogonal endomorphism. This study will be developed in Part II of this paper in the context of inertial referential frames.

4 Some meaningful outcomes.

Now, henceforth (for reason of what we exposed in the foregoing subsection), the space-time of the observer O_1, that is $\mathbb{L}_{O_1}$ works like the unique vectorial lorentzian space-time [13]. Therefore R **acts like an endomorphism into $\mathbb{L}_{O_1}$**. Therefore herein (F_1) and (F_2) work as the matrix components of fields F_1 and F_2 in the vectorial space $\mathbb{L}_{O_1}$ of O_1.

For convenience in the *Part 2* of this article we make $F \equiv F_1$; $F_2 \equiv R(F)$, $G_1 \equiv G_2 \equiv G$ and $\mathbb{L}_{O_1} \equiv \mathbb{L}_{O_2} \equiv \mathbb{L}_O$.

Anyway it is worthwhile to highlight that in this context O_1 is the observer that drives the observation.

[12] This proposition is subordinated to different considerations, for example the existence of an affine space in which $\mathbb{L}_{O_1}$ and $\mathbb{L}_{O_2}$ are embedded (without thinking into fibre bundle structures). Pro temp we hold the concept of affine space. Abiding by classic theory, somehow it involves an affine space structure, even if is questionable if it is about a movement (translation+rotation) or in general a bijection , or other kind of transformations (see [1].)

[13] That would be from the standpoint of observer O_1. In this way we can use the mathematical tools of the endomorphism.

4.1 The constancy of velocity of the light.

The orthogonal endomorphism R preserves the metric G.

In the minkowskian space-time the interval of universe defined by G in a inertial reference frame is

$$s^2 = -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$$

Here x^0 is a time dependent magnitude with the dimension of space. For this reason must be $x^0 = ct$ where t is the time measured by the observer, (into this structure of G, t works like a coordinate) and c is a coefficient.

Then the interval of universe is

$$s^2 = -(ct)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$$

Its metric is invariant under orthogonal transformation. Thereby the coefficient c is to be a dimensional physical constant, with the dimension of velocity, that remains constant in this orthogonal transformation.

It is not hard to see that c **is the light velocity** reasoning in the same way that in classical special relativity.

That is c**, the velocity of the light, remains constant in Lorentz transformations.** Or rather c **is invariant in a change of inertial referential frames.**.

This orthogonal transformation involves the constancy of velocity of the light.

We attain this fact without establishing the principle of the constancy of velocity of light.

4.2 Reduction to basic relativity.

The theory here explained is reduced to classic theory of relativity making $O_2 \equiv F_0$. That is, the event observed is coupled with an observer (O_2). This would involve that the observed event is endowed with a reference frame.

This is the case in which an observer O_2 sends luminous signals to observer O_1 likewise the especial theory of relativity of Einstein.

There are other different ways to identify O_1, O_2 and F_0, but their analysis is beyond the scope of this paper. However it would be worthwhile to take them into account.

5 Conclusions.

The starting points of this article are

1-.From the beginning we agree to the minkowskian space-time structure that is the lorentzian vectorial space of signature $(-1, 1, 1, 1)$ on a real field.

2-.In this structure of space-time, in the basic relativity theory it is proved that the electromagnetic field is a skew adjoint second order tensor. Hereupon we give up some principles (constancy of the velocity of light, Maxwell equations, homogeneity and isotropy of space, body rigidity, timing of clocks etc...) and **we only admit the electromagnetic field with a tensorial skew-adjoint character in the context of the minkowskian space-time.**

3-.We set up two principles:

Principle of coupling of observers. It is based on the fact that there are two basis (for two observers O_1 and O_2) in such a way that with respect them the observations gotten are the same.

On these basis we prove that ***there is an orthogonal transformation (homomorphism R) between the space-times of both observers that preserves the tensorial skew-adjoint structure of electromagnetic field.***

Transporter principle. In the beginning (as long as reference frames are inertial) , the vectorial space-times of observers are embedded into an affine space-time. It means that virtually we can **move** the reference frame of observer O_2 to the origin of the reference frame of the observer O_1 [14]. In view of this context R **can act as an endomorphism into the space of the observer O_1.**

In this specific way, in the second part of this paper we are studying the main features of the Lorentz transformations reaching the basic outcomes gotten in the relativity theory (specifically Lorentz boost).

[14] Actually from our point of view it is possible to make generalizations to other suitable structures.

ANNEXES

A Coordinates transformation.

As we saw earlier we use the following way to denote coordinates components:

$$(\overrightarrow{\mathbf{X}}) = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$(\overrightarrow{\mathbf{X}})^t = \begin{pmatrix} x^0 & x^1 & x^2 & x^3 \end{pmatrix}$$

Let be now $(\overrightarrow{\mathbf{X_{O_1}}})$ and $(\overrightarrow{\mathbf{Y_{O_1}}})$ matrices coordinates measured by observer O_1 and $(\overrightarrow{\mathbf{X_{O_2}}})$ and $(\overrightarrow{\mathbf{Y_{O_2}}})$ matrices coordinates measured by observer O_2, both in connection with the event observed F_0.

$(F_{\mathbf{O_1}})$ and $(F_{\mathbf{O_2}})$ are the components matrices of endomorphisms associated with fields observed by O_1 and O_2.

$(F_{\mathbf{O_1}})$ **and** $(F_{\mathbf{O_2}})$ **have nothing to do with coordinates (that is with** $(\overrightarrow{\mathbf{X_{O_1}}})$ **and** $(\overrightarrow{\mathbf{Y_{O_1}}})$ **matrices)** because we deem they are endomorphisms.

$$(\overrightarrow{\mathbf{Y_{O_1}}})^t = (\overrightarrow{\mathbf{X_{O_1}}})^t (F_{\mathbf{O_1}})$$

$$(\overrightarrow{\mathbf{Y_{O_2}}})^t = (\overrightarrow{\mathbf{X_{O_2}}})^t (F_{\mathbf{O_2}}) = (\overrightarrow{\mathbf{X_{O_2}}})^t (R)(F_{\mathbf{O_1}})(R^{-1})$$

$$(\overrightarrow{\mathbf{Y_{O_2}}})^t (R) - (\overrightarrow{\mathbf{Y_{O_1}}})^t = ((\overrightarrow{\mathbf{X_{O_2}}})^t R - (\overrightarrow{\mathbf{X_{O_1}}})^t)(F_{\mathbf{O_1}})$$

Because of $(F_{\mathbf{O_1}})$ is independent of coordinates (as we saw before) we attain

$$\boxed{(\overrightarrow{\mathbf{Y_{O_1}}})^t = (\overrightarrow{\mathbf{Y_{O_2}}})^t (R)}$$

$$\boxed{(\overrightarrow{\mathbf{X_{O_1}}})^t = (\overrightarrow{\mathbf{X_{O_2}}})^t (R)}$$

This outcome connect coordinates of observers O_1 and O_2.

B Notations, symbols and terminology

Vectors are symbolized with over right arrow.

Tensors and endomorphisms stand for bold or normal uppercase letters.

The matrix of components of an endomorphism, tensor, etc.. is shown closing inside parenthesis the symbol of this endomorphism, tensor, etc.. . For example $(\mathbf{T})$ stands for the matrix of components of $\mathbf{T}$. $(\mathbf{g}_{\alpha\beta})$ is a matrix which elements are $\mathbf{g}_{\alpha\beta}$.

However for convenience we omit parenthesis when specified in order for using symbols more easily.

The two vectors scalar product $\overrightarrow{\mathbf{x}}$ and $\overrightarrow{\mathbf{y}}$ is symbolized by $\mathbf{G}(\overrightarrow{\mathbf{x}},\overrightarrow{\mathbf{y}})$ where $\mathbf{G}$ is the metric tensor. Also is symbolized by $\overrightarrow{\mathbf{x}}.\overrightarrow{\mathbf{y}}$.

Subscripts are symbolized by lower case greek or latin letters, saving λ and μ that are used to denote invariants.

Usually E_2 symbolize a 2-dim euclidean space, L_2 a 2-dim vectorial lorentzian space and L_n a n-dim vectorial lorentzian space. Meanwhile we do not know if the space is lorentzian L_n or euclidean E_n, we symbolize these spaces with symbol $\mathbb{L}_n$ In general if it is not established if the space is lorentzian or eucledian we will use the blackboard bold letter [15] to represent the space.

$T^\sharp$ is de G-adjoint endomorphism of T. T^t is the transposed endomorphism of T. In regard to the called *endomorphism associated to a tensor* it is necessary to make clear that the components of the mentioned endomorphism are those of the mixed components of the tensor.

[15] For example L in blackboard bold letter is $\mathbb{L}$

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[6] Marsall C. Pease III; Methods of Matrix Algebra; Academic Press 1965; pag 223 and next; Adjoint operator . This book works on covariant coordenates. Here we work on contravariant coordinates. Therefore here $F_i^\sharp = GF_i^tG^{-1}$ $i = 1, 2$. ; In general we work on contravariant coordenates unless otherwise specified. It is a theorical background essential.

[7] Changrim Jang , Philips E. Parker; Skewadjoint Operators on Pseudoeuclidean Spaces; arXiv:math/0302030