Insights into the Theory of Relativity. Part III. Dynamic Relativist. 2.-Effect of the metric of einstenian space-time in the matter and energy equivalence.

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Abstract

We highlight that the classical prove of the equivalence between matter and energy, arise from the fact of the existence of energy, force, and acceleration into the context of an inertial referential frame. It shows up a contradiction with the concept of inertial referential frame.

Therefore, the equivalence between matter and energy, and the variation of the classical mass with velocity , must be developed taking into account the metric of the einstenian space. This involves gravitational field and stress-energy tensor (namely, at least, energy and forces in the classical sense).

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Contents

1 Introduction

In this article we shall deal with equivalence between matter and energy and the variation of the mass with velocity.

We can see the prove of the equivalence between mass and energy in the classic theory in [3] [1]. We highlight that acceleration, forces, etc... come up in the deduction of the equivalence between matter and energy into the context of inertial reference frames, in the special theory of relativity.

Thereby we consider that, in the especial theory of relativity, we have a string of events between an initial equilibrium rest microstate and a final equilibrium rest microstate, in the analysis of the dynamic relativist (where can be forces and accelerations, between the initial and final state).

The purpose of this article is to develop the theme of the transformation between mass and energy *taking into account the einstenian spaces with metric.*

2 Generalized magnitudes in the minkowskian spaces.

In the special theory of relativity, the physical magnitudes are to be tensorial magnitudes in the minkowskian space-time.

[1]This prove appear in many books and articles about the theory of relativity, without thinking into the original articles of Einstein.

Into the context of this minkowskian space L_4, these tensorial magnitudes have a physical meaning in the theory of relativity,

By the way, all the magnitudes defined into L_4, are called *generalized magnitudes*.

These *generalized magnitudes* only have physical sense into L_4, because of they have to be defined into the space-time.

Generalized magnitudes are in connection with *non relativistic magnitudes* of classical physics.

The *generalized magnitudes* act in the space-time. The *non relativistic magnitudes* act into the galilean context.

The *non relativistic magnitudes* are fitted to the spatial or temporal components of the *generalized magnitudes*.

3 The equivalence matter energy in einstenian space-times.

In the relativity theory physical magnitudes have to be defined in the 4-DIM einstenian space too. We deem these magnitudes have a tensorial structure, that is, *generalized tensor magnitudes* , in the cited space agree with the *relativity principle*. The physical magnitudes of classical non relativistic mechanics are organized into the space or time components of the generalized tensor components.

In this article, we are dealing with an einstenian space-time [2] , that is a 4-dim space-time endowed of a metric 1:

$$ds^2 = -g_{00}(dx^0)^2 + 2\sum_{i=1}^{i=3} g_{0i}dx^0dx^i + \sum_{i,j=1}^{3} g_{ij}dx^idx^j \tag{1}$$

ds is the *interval of universe*. We have $dx^0 = cdt$. Then:

$$ds^2 = -g_{00}c^2dt^2 + 2\sum_{i=1}^{i=3} g_{0i}cdtdx^i + \sum_{i,j=1}^{3} g_{ij}dx^idx^j$$

Let be now τ the *proper time*, that is the time measured by the observer that is moving with the reference frame.

$$ds^2 = -g_{00}c^2dt^2 + 2\sum_{i=1}^{i=3} g_{0i}cdtdx^i + \sum_{i,j=1}^{3} g_{i,j}dx^idx^j = -c^2d\tau^2$$

[2] Einstenian space-time is boiled down to minkowskian space-time when the metric is transformed into a minkowskian metric

Therefore

$$d\tau = dt\sqrt{g_{00} - 2\frac{g_{0i}v^i}{c} - \frac{v^2}{c^2}} \quad \text{i=1,3}$$

where

$$\overrightarrow{v} = (\frac{dx^1}{dt}, \frac{dx^2}{dt}, \frac{dx^3}{dt}) \quad ; \quad v = \sqrt{\overrightarrow{v}^2}$$

$$\boxed{\gamma = \sqrt{g_{00} - 2\frac{g_{0i}v^i}{c} - \frac{v^2}{c^2}} \;\; ; \text{i=1,3}}$$

$$d\tau = \gamma dt$$

We continue with a reasoning likewise [3] .

$$\overrightarrow{U} = \frac{d\overrightarrow{x}}{d\tau} = (\frac{dx^0}{d\tau}, \frac{dx^1}{d\tau}, \frac{dx^2}{d\tau}, \frac{dx^3}{d\tau})$$

where $\overrightarrow{U}$ is the the *generalized velocity*
We have

$$\overrightarrow{U} = \frac{d\overrightarrow{x}}{d\tau} = (\frac{c}{\gamma}, \frac{dx^1}{\gamma dt}, \frac{dx^2}{\gamma dt}, \frac{dx^3}{\gamma dt})$$

namely

$$\overrightarrow{U} = (\frac{c}{\gamma}, \frac{\overrightarrow{v}}{\gamma})$$

Let be now $\overrightarrow{\Pi}$, the *generalized linear momentum.*
Therefore

$$\overrightarrow{\Pi} = m_0\overrightarrow{U} = (\frac{m_0}{\gamma}c, \frac{m_0}{\gamma}\overrightarrow{v}) = (mc, m\overrightarrow{v})$$

where m_0 is the mass at rest (generalized mass) and m has to be the mass of the matter in the classic theory.

$$\boxed{m = \frac{m_0}{\sqrt{g_{00} - \frac{2g_{0i}v^i}{c} - \frac{v^2}{c^2}}} \;\; ; \text{i=1,3}}$$

The *force generalized* $\overrightarrow{\Phi}$ is:

$$\overrightarrow{\Phi} = \frac{d\overrightarrow{\Pi}}{d\tau} = \frac{1}{\gamma}(\frac{d(mc)}{dt}, \frac{d(m\overrightarrow{v})}{dt})$$

Taking into account that $\overrightarrow{U^2} = -c^2$ it is verified

$$\overrightarrow{\Pi}^2 = m_0^2.\overrightarrow{U}^2 = -c^2 m_0^2$$

Then

$$\overrightarrow{\Pi}.\frac{d\overrightarrow{\Pi}}{d\tau} = 0 \quad \text{therefore} \quad \overrightarrow{\Pi}.\overrightarrow{\Phi} = 0$$

$$\frac{1}{\gamma}(\frac{d(mc)}{dt}, \frac{d(m\overrightarrow{v})}{dt}).(mc, m\overrightarrow{v}) = 0$$

It is not hard to see

$$g_{ij} m v^i \frac{d(mv^j)}{dt} - g_{00} mc\frac{d(mc)}{dt} + g_{0j} mc\frac{d(mv^j)}{dt} + g_{i0}\frac{mv^i d(mc)}{dt} = 0$$

$$\overrightarrow{v}.\overrightarrow{f} = \frac{dE}{dt} = g_{00}\frac{d(mc^2)}{dt} - g_{i0}(c\frac{dp^i}{dt} + v^i\frac{d(mc)}{dt})$$

We find the formula of transformation between mass and energy, taking in account reference frames not at rest.

$$\boxed{\delta E = g_{00}\delta(mc^2) - g_{i0}(c\delta p^i + v^i\delta(mc))}$$

$\overrightarrow{E}$, $\overrightarrow{f}$ and $\overrightarrow{p}$ are the *energy*, *force* and *linear momentum* in the classical basics theory.

In the absence of gravitation, or forces (stress-energy tensor vanishing, and $g_{\alpha\beta} = 0$ if $\alpha \neq \beta$, $g_{00} = 1$;$g_{ii} = 1$; $i = 1, 3$) ,the formula of equivalence between mass and energy is:

$$\boxed{\delta E = \delta(mc^2)}$$

This is the famous formula of Einstein for inertial reference frames in the special relativity theory.

A general way to highlight the negative terms is:

$$\boxed{\delta E = \delta(mc^2)(g_{00} - \frac{g_{i0}v^i}{c}) - cg_{i0}f^i\delta t}$$

The negative terms point out cases where an increment positif of mass is transformed into an increment negative of energy (non relativistic). Therefore, in these cases the energy goes down when mass is increased.

4 Application to solar system : earth

In the solar system, using the Schwarzschild model, we calculate how influences the gravity of sun on the transformation of matter in energy near the earth. In *figure 1*, it is shown this influence

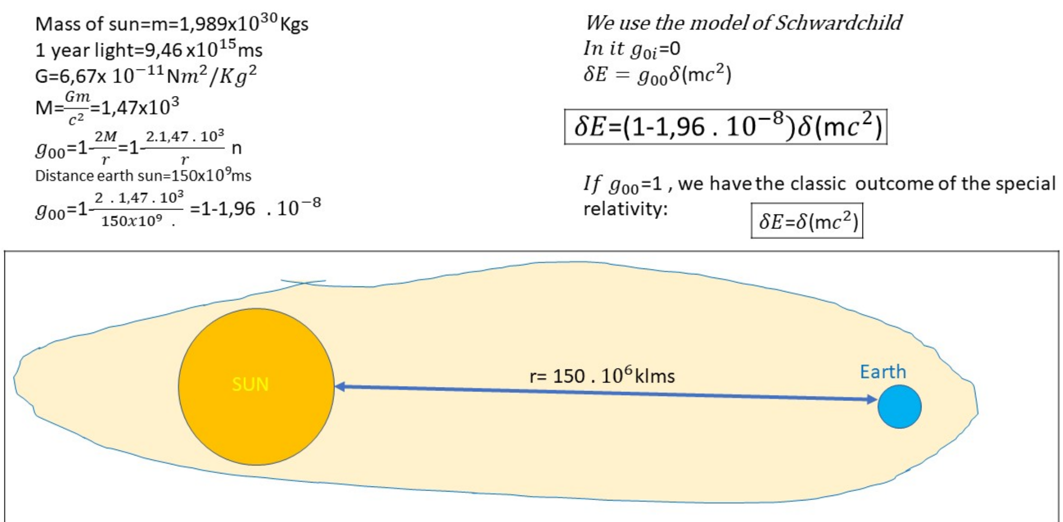

Fig 1

In this way it is reported the transformation of matter in energy. It is reduced in the order of 10^{-8}, that is

$$\delta E = (1 - 1,96.10^{-8})\delta(mc^2)$$

Likely, the influence of the gravitation of sun in the transformation matter-energy, can be neglected near the earth.

5 Conclusions

At first glance, the influence of metric of einstenian space-times on mass energy equivalence, can be neglected.

However, into some critical contexts, (as in the proximity of black holes), the influence of the metric of einstenian space-times, can be extremely important.In my opinion, it is a theme of study.

I deem, other interesting matter of study is the action of the negative terms reported in the previous section.

The developments here reported, are full compatible with our articles [1], [2], [3], [4].

6 Notations, symbols and terminology

Vectors are symbolized with over right arrow.

Tensors and endomorphisms stand for bold or normal uppercase letters.

The matrix of components of an endomorphism, tensor, etc.. is shown closing inside parenthesis the symbol of this endomorphism, tensor, etc.. . For example **(T)** stands for the matrix of components of **T**. $(\mathbf{g}_{\alpha\beta})$ is a matrix which elements are $\mathbf{g}_{\alpha\beta}$.

However for convenience we omit parenthesis when specified in order for using symbols more easily.

The two vectors scalar product $\overrightarrow{\mathbf{x}}$ and $\overrightarrow{\mathbf{y}}$ is symbolized by $\mathbf{G}(\overrightarrow{\mathbf{x}}, \overrightarrow{\mathbf{y}})$ where $\mathbf{G}$ is the metric tensor. Also is symbolized by $\overrightarrow{\mathbf{x}}.\overrightarrow{\mathbf{y}}$.

Subscripts are symbolized by lower case greek or latin letters, saving λ and μ that are used to denote invariants.

Usually E_2 symbolize a 2-dim euclidean space, L_2 a 2-dim vectorial lorentzian space and L_n a n-dim vectorial lorentzian space. Meanwhile we do not know if the space is lorentzian L_n or euclidean E_n, we symbolize these spaces with symbol $\mathbb{L}_n$ In general if it is not established if the space is lorentzian or eucledian we will use the blackboard bold letter [3] to represent the space.

$T^\sharp$ is de G-adjoint endomorphism of T. T^t is the transposed endomorphism of T. In regard to the called *endomorphism associated to a tensor* it is necessary to make clear that the components of the mentioned endomorphism are those of the mixed components of the tensor.

[3] For example L in blackboard bold letter is $\mathbb{L}$

References

[1] **F.Sánchez; Structures of the Skew-adjoint Endomorphisms and Some Peculiarities of Electromagnetic Field.(2014). relativityworkshop.com**.It is the theorical background necessary to understand the part II.

[2] **F.Sánchez; Insights into theory of relativity. Part I. Critical approachs. Basics principles and starting points (2017). relativityworkshop.com**

[3] **F.Sánchez; Insights into theory of relativity. Part II. Loretz Transformation (2018). relativityworkshop.com**

[4] **F.Sánchez; Insights into theory of relativity. Part III.Dynamic Relativist.1.-Matter and Energy Equivalence. (2020). relativityworkshop.com**

[5] **A. A. Logunov ; Lectures on Relativity Theory and Gravitation. URSS Publishers 1998; spanish edition). Lectures 5 and 6. Highlight the relevance that Poincaré found out in the development of special relativity. It is really important to single out that the basics of classic Theory of Relativity are in the transformations that leave invariant the Maxwell equations.**

[6] **Gregory L. Naber; The geometry of Minkowski spacetime ;(Springer Verlag 1982). It is a full analysis of the skew-adjoint nature of electromagnetic field**

[7] **Roger A. Horn; Matrix Analysis;(Cambridge University Press 1985). It is an appealing text about matrix analysis really useful as theorical background here.**

ERRATUM

Page 4; line 4. Says $v = \sqrt{\vec{v}^2}$; must say: $v^2 = \mathbf{g}_{\alpha\beta} v^\alpha v^\beta$; $\alpha, \beta = 1, 3$